The Central Executive of Working Memory and its Inhibitory Rôle in Mental Arithmetic Division

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ABSTRACT

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A four experiment investigation was carried out into two types of inhibitory control and whether or not they were involved in mental arithmetic division. Previous research had either focussed on the central executive component of working memory as a single entity or split this component into response selection, memory updating, input monitoring and inhibition and has examined how these components relate to addition, subtraction and multiplication. The first two experiments reported here represent an attempt to split inhibition into two separate abilities, namely prepotent response inhibition and resistance to distracter interference and investigate how they might relate to complex division. The Two experiments were loosely based on the Stop-Signal Task and the Eriksen Flanker Task, respectively. Experiment One followed a 3x3x3 factorial design whereas Experiment Two was a 3x2 design. All factors were varied entirely within-subjects. The results suggested a major rôle in mental division for both types of inhibition and some results were unexpected. The results of Experiment Two suggested a speed-accuracy trade-off as an effect of the flanker digits.

Experiment Three was designed to extract the short division procedure from the division process. Experiment Four was created to extract the carrying procedure. The results suggested a rôle for both prepotent response inhibition and resistance to distracter interference in monitoring the procedural aspects of complex division. Only resistance to distracter interference had a rôle in monitoring magnitudes of short-division and carry-values. The speed-accuracy trade-off apparent in Experiment Two may, at least in part, be as a result of the involvement of resistance to distracter interference in preventing interference with regard to carry procedures and the value of the carry. The results were discussed, amongst other aspects, in terms of their relationship to conflict monitoring theory and dual mechanism of control theory (Braver *et al*, 2007, 2009; Botvinick *et al*, 2001) and a two-channel system of inhibition was proposed, resistance to distracter interference being a proactive subcomponent and prepotent response inhibition being reactive.

List of Contents

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CHAPTER ONE

INTRODUCTION

1.1 Arithmetic Difficulties

The most severe cases of arithmetic difficulties include persons with either acquired or developmental acalculia. The most notable difficulties that were present were those of quantity manipulation (Dehaene, 1992; Kaufman, 2002; Kaufmann, Lochy, Drexler & Semenza, 2004; McCloskey, 1992; McCloskey, Caramazza, & Basili, 1985; Warrington, 1982), such as carrying, borrowing, decomposition and the processing of operation symbols. It has been suggested by these authors that, in many cases, poor executive functioning may be a causal factor. Such poor executive functioning will result in difficulty keeping track of carry operations, difficulty retrieving number facts from the long term memory [although this may also be as a result of incomplete memory for times-tables] (Kaufman *et al*, 2004), poor inhibition of incorrect responses and poor retrieval of appropriate methods. The present study is designed to examine one of these executive functions, that of inhibition.

Whether poor inhibitory control is a causal factor in arithmetic difficulties is a controversial matter. Barrouillet, Fayol & Lathulière (1997) studied teenagers with arithmetic difficulties and suggested that at the beginning of the multiplication process a number of possible responses are cognitively processed, the incorrect ones of which need to be inhibited. If a teenager has poor inhibitory control, these incorrect responses are more likely. Teenagers with mathematical difficulties, according to Barrouillet *et al* (1997), may have problems triggering inhibitory control and this may be a causal factor of the problem. More recent studies, however, cast doubt on poor inhibitory control being a causal factor of arithmetic difficulties. These studies compared test scores of inhibitory control of children with arithmetic difficulties with those of children without such difficulties and found no significant difference in inhibitory control (Censabella & Noël, 2004, 2005, 2008). These authors suggested limited working memory capacity as a more likely factor, causing confusion between closely related number facts, i.e., a problem such as 8 x 3 might activate 21 and 27 as well as 24. The former two responses need to be inhibited (e.g., Campbell, 1987). What cannot be made directly clear from comparing such scores is whether or not inhibitory control takes place during arithmetic processing. This is an issue the present study is designed to address, within a behavioural paradigm.

1.2 Numerical Cognition and Methods for Solving Arithmetic Problems

Numerical Cognition refers to the recognition of numbers, quantity perception, and processing of calculations (McCloskey, 1992; McCloskey, Caramazza, & Basili, 1985). McCloskey (1992) and Dehaene (1992) each proposed their own models of numerical processing. McCloskey's model contains subsystems for symbolic and number comprehension, internal representation, calculation procedures and symbolic and verbal number comprehension. In this model, numerical inputs are converted via a numerical comprehension system into semantic representations ready to be used in cognitive processes such as calculation procedures (McCloskey, 1992). The *Triple Code Model* proposed by Dehaene (1992) is a little more detailed and also consists of three components: the Analogue Magnitude Representation, the Visual Arabic Number Form, and the Auditory Verbal Word Frame. Taking these components respectively, the analogue magnitude representation is based on a number line ranging from zero to infinity (so 14 might be represented as: ___10 ↓ ___20___ , i.e. a number between 10 and 20) and may be used for comparison and approximation. The visual Arabic number form is based on Arabic symbol format and may be involved in multi-digit problem solving. The auditory verbal word frame is based on the orthographic written format of a number and processes written and spoken representations of number as well as counting, and multiplication and addition facts. With regard to the present study, what is of most interest is the subsystem for calculation procedures in McCloskey's (1992) model and the Arabic number form in Dehaene's (1992) model; it is calculation with regard to division that will dominate.

Different people use a range of differing methods when solving arithmetic problems; this was a potential problem in the present study. Just to examine some of them briefly, there are a number of studies that have researched arithmetic methods in children and adults, some of which are taught or encouraged and others of which are self-developed. Beishuizen, Van Putten and Van Mulken (1997) studied methods used by third grade (year four) children. Two different methods were researched: decomposition, that is first splitting numbers into units, decades and so on to make the numbers easier to deal with; and counting by tens, up or down from the first un-split number and then adding (or subtracting) appropriate units.

Much work on children's arithmetic methodology was carried out recently, in Australia. Heirdsfield (2002) suggested a list of different methods for various arithmetic procedures following a series of self-reporting interviews with children (aged 8 – 10 years). Regarding addition and subtraction, four categories of methodology were proposed: separation, aggregation (counting forwards or backwards), approximating prior to calculating the exact response and mental imagery of pencil and paper methods. Heirdsfield and Cooper (1997) suggested that if aggregation is employed, then a lighter load is placed on working memory. Heirdsfield and her colleagues do consistently maintain that the least efficient method of mental arithmetic is to use the mental facsimile of pencil and paper methods owing to its rigidity (Heirdsfield, 2002; Heirdsfield & Cooper, 1997, 2002).

With respect to adults, LeFevre and colleagues (2006) reported on certain methods used for both simple and complex subtraction, using a self-reporting method. The methods reported were retrieval, sequencing and various transformational methods. There was a tendency for participants to use retrieval more often for easy problems and some type of procedural method for harder problems (LeFevre, DeStefano, Penner-Wilger & Daley, 2006). LeFevre and co-researchers (2003) reported that for problems such as 16 – 9 and 37 – 9, where borrowing is a requirement, decomposition was a popular choice of procedure (LeFevre, Smith-Chant, Hiscock, Daley & Morris, 2003). Ultimately, the method used depended upon the construction of the problem and individual differences – this applied to both children and adults (LeFevre *et al*, 2003; LeFevre *et al*, 2006). With regard to adults, it also depends on the amount of practice they have had using particular methods and how much proficiency they have gained (Ischebeck *et al*, 2008; Miller, Perlmutter & Keating,1984). If consistency of method was to be attained in the present study then a single calculation method needed to be encouraged; this will come to light in the next section.

1.3 Complex Division

As well as the wide variety of methodologies that are used to solve arithmetic problems, it was noticeable that there is a lack of literature pertaining to division; this is one of the gaps the present study is designed to fill. Focus has tended to be on simple rather than complex division (e.g., Robinson & Ninowski, 2003; Robinson, Arbuthnott & Gibbons, 2002; Imbo & Vandierendonck, 2007). Simple division problems are those that comprise a single or double digit dividend and a single digit divisor, e.g., 9 ÷ 3 or 25 ÷ 5, whereas complex division problems contain a dividend with three or more digits and a single digit divisor, e.g., 1296 ÷ 4. It is the latter type of problem that will be of interest in the present study. Complex division should not be confused with long division, e.g., 1675 ÷ 25. Obviously, short division is far less cognitively demanding than complex division and is likely to involve direct retrieval of responses from LTM (e.g., Campbell, 1999; Campbell & Alberts, 2010). On the other hand complex division is far more proceduralised and will involve multiple calculation stages such as multiple short divisions and carrying, hence drawing upon far more working memory resources (*cf.*, Fürst & Hitch, 2000; Imbo, Vandierendonck & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2002). As an arithmetic operation, division is not used as much as multiplication and is therefore less well practised (Robinson & Ninowski, 2003) and tends to be the weakest of the four operations (Robinson *et al*, 2002). It has been suggested earlier that participants were likely to use a variety of methods to solve complex division problems (e.g., Ischebeck, Zamarian, Schocke, & Delazer, 2008). Participants in the present study were therefore encouraged to use the following method.

For example problem such as 1269 ÷ 3 was set out as follows:

$$\underline{1269}$$
$$\underline{3}$$

The response was typed left to right on the number keys of the computer; the procedure encouraged was: "Three into one won't go, therefore treat the 1 and 2 as 12; three into twelve goes in four times; type 4; three into 6 goes twice, type 2; finally three into 9 goes three times, type 3; the answer is 423". This is an example of a problem with no remainder carrying and is simply a series of short division procedures. The procedure with regard to problems demanding carrying is explained in Chapter Three (Experiment One).

1.4 The Present Study

Four Experiments were implemented over the course of the present study with the overall aim of examining the relationship between complex division, and prepotent response inhibition and resistance to distracter interference. They were designed to answer two broad questions: (1) Are prepotent response inhibition and resistance to distracter interference, as types of inhibition involved in the processing of complex division problems? Are there any specific procedures, within the complex division process that benefit from the utilisation of these particular types of inhibition? It has already been suggested that there is controversy with regard to whether or not inhibitory control is a causal factor of arithmetic difficulties (Barrouillet *et al* 1997; Censabella & Noël, 2004, 2005, 2008). What is not clear is whether or not inhibition is used as part of the processing system when solving arithmetic problems.

Prepotent response inhibition and resistance to distracter interference are two subcomponents of working memory. Very briefly, the model of working memory upon which this study is based the four component model of Baddeley (2000). The four components comprise three slave systems controlled by an attention controlling central executive. The slave systems comprise the phonological loop for processing phonological and auditory information, the visuo-spatial sketch pad for processing visual and spatial information, and the episodic buffer which acts as an interface between the other two slave systems and information from the LTM (Baddeley, 2000). The slave systems are controlled by the central executive which allocates resources to the slave systems according to the complexity of the task being undertaken and also provides a link to LTM (Baddeley, 2000). It had been proposed that the central executive could be fractionated into a dual task coordinator (input monitoring), a link to LTM (memory updating), a response selection mechanism, and a mechanism to filter unwanted information from a particular task (stimulus inhibition) (Baddeley, 1996). It was later proposed that stimulus inhibition could be separated into three abilities: prepotent response inhibition, for resisting dominant tendencies; resistance to proactive interference for filtering previously useful information that was no longer needed; and resistance to distracter interference for filtering external intrusions (Friedman & Miyake, 2004).

Experiment One focussed on prepotent response inhibition and its rôle with regard to the whole complex division process with the aim of answering the question: Is prepotent response inhibition employed by the cognitive processing system when solving complex division problems? Experiment Two followed this up but had the aim of answering the same question but with regard to the rôle of resistance to distracter interference. Experiment Three represents an attempt to separate the short division procedure to discover whether or not prepotent response inhibition or resistance to distracter interference have an effect on this particular procedure. Experiment Four was designed to answer the same question but with regard to the carrying procedure. A further aim of the present study was to examine the processes of prepotent response inhibition and resistance to distracter interference. Friedman and Miyake (2004) suggested these types of inhibition were part of a shared mechanism to filter unwanted intrusions from the task being undertaken, a dual channel inhibitory system was proposed, as a result of the present study. It was further proposed that a small contribution could be made to enhancing Baddeley's (2000) model of working memory.

CHAPTER TWO

The Central Executive of Working Memory and its Inhibitory Rôle in Mental Arithmetic Division

Review of Literature

2.1: Introductory Remarks

Arithmetic division tends to be regarded as the fourth arithmetic operation, after addition, subtraction and multiplication. Fourth, because it tends to be the last operation studied in school and, even in adulthood, has been found to be the least proficiently handled out of the four operations of arithmetic (Robinson, Arbuthnott & Gibbons, 2002). It is probably the case that, owing to this, there is a general lack of literature pertaining to complex division and it is hoped that the present study will go part way to reversing this. By complex division, what is meant is a multi-digit dividend divided by a single-digit divisor (e.g., 1655 ÷ 5). This is not to be confused with simple division (e.g., 9 ÷ 3) or long division (e.g., 2575 ÷ 25). For methodological reasons, the 'top-heavy fraction' format was used throughout the present study, e.g.,

$$\frac{1465}{5}$$

The concept of working memory has developed considerably since it was first proposed by Baddeley & Hitch (1974), from the three component model through the fractionation of the central executive to the addition of the episodic buffer (Baddeley, 2000). Although other models of working memory exist (e.g., Cowan, 1988; Demetriou, Christou, Spanoudis & Platsidou, 2002) it was decided, for the purpose of the present study to use Baddeley's (2000) model as the basis for investigation. There is much agreement that Baddeley's Model, being a multi-component system can be separated and the separate components loaded and, as a model, it has helped and continues to be a much recognised guide for research into specific cognitive processes (Andrade, 2001).

What is regarded as the most important component of working memory is the central executive. The rôle of the central executive is in attentional processes such as decision making, reasoning, language comprehension, the allocation of the relevant proportion of visual or spatial resources needed to process a particular task, the inhibition of irrelevant information, and to transfer information to the long-term memory (Ashcraft, 1994; Baddeley, 1997; Logie, 1995). In addition to being fractionated (Baddeley, 1996) the central executive has caused controversy with regard to whether it is just a homunculus (Parkin, 1998) or might operate more like an executive committee (Baddeley, 2003). Starting points for such an executive committee are the four fractionated sub-components, namely input monitoring, memory updating, response selection and inhibition (Baddeley, 1996) that are reviewed later in this Chapter. Related to this attentional system are further theories of attentional control from controlled attention (Engle, Tuholski, Laughlin, & Conway, 1999), through conflict monitoring theory (Botvinick, Braver, Barch, Carter, & Cohen, 2001), lateral inhibition (Verguts & Fias, 2005), and adaption by binding theory (Verguts & Notebaert, 2009) to dual mechanism of control theory (Braver, Gray, & Burgess, 2007; Braver, Paxton, Locke, & Barch, 2009).

The subcomponent highlighted in the present study, *inhibition*, is thought to aid mental arithmetic by filtering inappropriate methods such as a strong tendency to solve multi-digit problems that require carrying without doing so (Fürst & Hitch, 2000; Imbo, Vandierendonck & Vergauwe, 2007) or to suppress closely related yet incorrect number facts from long-term memory [LTM] (Campbell & Clarke, 1989). Although Baddeley (1996) proposed inhibition as a single entity, later research has suggested that inhibition can be separated into three types: prepotent response inhibition, resistance to distracter interference and resistance to proactive interference (Friedman & Miyake, 2004); as will be explained later, for methodological reasons, the first two types will be the focus of the present study. *Prepotent response inhibition* (PRI) is charged with responsibility to suppress dominant or automatic responses whereas *resistance to distracter interference* (RDI) is an executive ability with a responsibility to suppress unwanted external intrusions from working memory during task processing (Friedman & Miyake, 2004). What is not clear, in the literature, is the type or types of inhibition employed by the cognitive processing system for filtration of inappropriate methods or incorrect responses when solving complex division problems. A purpose of the present study is to at least begin to address this issue.

A further purpose of the present study is not only to apply a quasi-dual-task behavioural paradigm to the study of executive abilities and arithmetic but also to attempt to address the issue of impurity in terms of activities designed to tap executive abilities. Miyake *et al* (2000) suggest there is a possibility that memory updating requires inhibition in order to work properly, for example, to inhibit inappropriate responses whilst updating a series of intermediate responses in an arithmetic problem. Hence one activity designed to assess a particular executive function may also assess another. For example, from a behavioural perspective, Deschuyteneer *et al* (2006) used the '1 back two-choice' reaction time task to disturb memory updating – an activity that might also tap interference control (RDI). This suggests difficulty in finding activities that will load only *one* executive function and represents a challenge that the present study attempts to alleviate, albeit to a greater or lesser extent.

2:2 Cognitive Arithmetic Processing

It has been accepted for many decades that arithmetic competency is extremely important, for example to check change after a purchase, and more pertinent to the present study to calculate the number of items or different items can be bought within a specified budget. Such calculations are more pertinent to the present study because they are likely to involve division When solving arithmetic problems, a number of cognitive procedures are executed, beginning with the recognition and perception of quantities, through the calculation process, to the eventual response (McCloskey, 1992; McCloskey, Caramazza, & Basili, 1985).

McCloskey *et al* (1985), using evidence from the study of a patient with acquired dyscalculia, suggested that when carrying out a calculation, one puts into operation three distinct procedures, as illustrated in Figure 2.1. First, the numbers or quantities have to be comprehended, second a calculation or retrieval procedure is executed, and third, the answer is produced. These were proposed as distinct procedures following evidence from the study of a patient who could present the correct answer from a multiple choice list but who would write down or say incorrect answers to problems when no multiple choice lists were provided, indicating intact number recognition and calculation but a dysfunctional production system. McCloskey *et al* also proposed separate representations of the above comprehension and production mechanisms.

Comprehension → Calculation → Production

Figure 2.1. A Simplified Diagram of the Problem Solving Procedure (based on McCloskey *et al* (1985), p. 173).

Further examination of the literature reveals not only separable numerical processes but also separable calculation techniques. Warrington (1982) provides a case study of a patient with left posterior parieto-occipital lesions whose basic addition and subtraction processing latencies were both slow and variable; this was coupled with an above average error rate. Error rates increased with the size of the minimum numerical coefficient (e.g., 19 – 3 was more error susceptible than 19 – 1) and multiplication was slower than for control subjects. However, estimation of quantity was unimpaired and number-size judgement was only marginally slower than average. Below average scores were evident for the four rules of arithmetic (where exact quantity facts are normally expected) but higher level arithmetic skills such as fractions, conversions, approximation, rules for expressions, ratio, proportion and percentage were slightly above average. It was proposed that exact quantity facts, approximate quantity facts, general arithmetic processing and arithmetic computation were all separable (Warrington, 1982).

From the point of view of conducting a behavioural study of mental arithmetic, this raises questions with regard to the possible separabilty of arithmetic procedures such as retrieval of number facts from LTM, maintaining intermediate results and carrying. McCloskey *et al* (1985) listed arithmetic problems encountered by a number of patients: selective fact-retrieval deficits, where certain facts were retrievable but not others; place value deficits, such as failure to carry digits when multiplying; inconsistent carrying, including carrying an inappropriate number and not carrying when it was required; and confusion of steps, such as writing a double digit number where a single digit should be inserted. They also cited patients with selective impairments with respect to different mathematical operations. It was therefore proposed that there may be separate cognitive arithmetic fact systems for each operation. Related to this, Dehaene & Cohen (1997) argued that multiplication and addition facts are mostly learned by rote methods whereas most subtraction and complex addition relies on semantic manipulation of numerical quantities. It was proposed that there were multiple cognitive routes for mental arithmetic: quantitative number processing and direct retrieval of rote verbal knowledge of tables (Dehaene & Cohen, 1997). The present study will attempt to separate two of the procedures mentioned above, namely carrying and short division; short division may be equated with direct retrieval of division facts from LTM. More will be stated about this in Chapters Five and Six. Meanwhile, what has been reviewed above is with reference to arithmetic procedures. Some models have greater specificity with regard to number facts and how they are accessed.

2.3 Models Containing Greater Specificity

Before turning to division, the focus of the present study, and because division is the inverse operation of multiplication, it is multiplication that will be examined next within the context of some later models of number processing. Some researchers have proposed more narrowly focussed cognitive arithmetical models with greater specificity. Verguts and Fias (2005) have suggested an interactive model for the retrieval of products in single digit multiplication. They proposed that the model consists of a semantic field containing an internally organised network of multiplication facts distributed on the basis of the size of the operands. For example, similar operands such as 7 x 7 and 6 x 7 are stored closely together; furthermore, it is assumed that for commutative problems (e.g., 3 x 8 and 8 x 3), only one product is represented. It is also assumed that commutative problems are represented in the form maximum x minimum (*cf.:* Butterworth, Zorzi, Girelli & Jonckheere, 2001). The semantic field could take the form of a rectangle (rather like a tables-chart) but, owing to commutativity, the upper right-hand section can be diagonally deleted to form a triangle. From this semantic field, activation is transferred to two decomposition fields, one representing tens (decades), the other, units. From the two decomposition fields, activation is transferred to a response field containing one 'cell' for each possible response (1 to 99); see Figure 2.2.

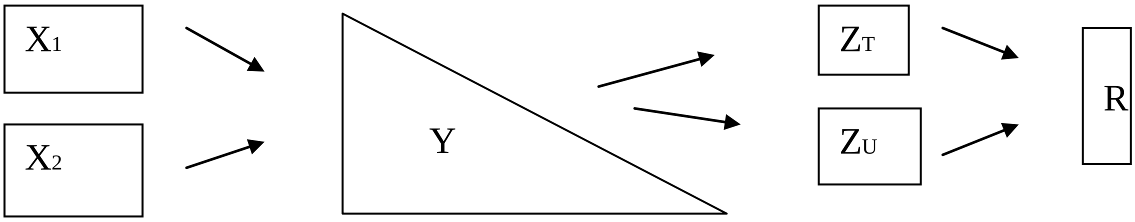

Figure 2.2. A Representation of the Connectionist Model of Retrieval in Single-Digit Multiplication (adapted from Verguts & Fias, 2005, p. 6)

Note: X_1 and X_2 are two operands, e.g.: 4 and 7. Y is the semantic field containing the network of facts which activates 7 x 4 = 28. Z_T is the tens (decades) field representing 2 (x 10) and Z_U is the units field representing 8. R is the response field, in this case, combining 20 (2 x 10) and 8 to represent 28.

To digress slightly, and because addition and multiplication are similar in that they are both commutative (i.e., 4 + 7 = 7 + 4), addition will be looked at briefly. Butterworth *et al* (2001) proposed the COMP model. They suggested that, when a problem, such as 2 + 4 is solved, these quantities are first identified. Next, they are compared and assigned maximum and minimum status and reordered, if necessary: in the case of 2 + 4, this will become 4 + 2. The sum is then retrieved from the cognitive addition table – again, because addition is commutative, this is just half a table in the form of a triangle (*cf.:* Verguts & Fias, 2005, re. multiplication). It was noted that in the number comparison task within this study, latencies were approximately 13 ms shorter if a problem was presented as 6 + 3 rather than 3 + 6.

According to these models, both addition and multiplication facts may be stored in a triangular cognitive semantic field consisting of an addition or multiplication chart cut diagonally in half. Skilled adults, very likely as part of the calculation process, retrieve number facts from the relevant semantic fields as part of solving simple arithmetic problems (Butterworth *et al,* 2001; McCloskey *et al,* 1985; Verguts & Fias, 2005). Domahs and colleagues provided both behavioural and EEG evidence of such semantic fields using verification tasks for simple multiplication (Domahs, Domahs, Schlesewsky, Ratinckx, Verguts, Willmes & Nuerk, 2007). Their findings suggested that decade consistent answers such as 7 x 4 = 24 took longer to reject than totally inconsistent answers, for example, 7 x 4 = 37. The notion forwarded here is important with respect to the present study in that, within the semantic field (tables-triangle), in order to verify a correct answer, the possible answers surrounding the correct response have to be *inhibited*. Possible answers completely unrelated to the correct one (7 x 4 cannot possibly be 37; 37 is prime) are inhibited far more easily than a possible response that belongs to the times-table of one of the operands, such as 7 x 4 = 24 (Domahs *et al*, 2007). This is comparable with the findings of Verguts & Fias (2005) who maintained that consistent neighbours led to more cooperation but inconsistent neighbours result in more competition, as referred to earlier.

Rickard (2005) revised a model of arithmetic fact representation that had previously been proposed. The Identical Elements (IE) model, first introduced in 1994, states that where a triplet of numbers (e.g., 5, 7, 35) are related by multiplication and division, three independent fact representations stored in the memory: 5 x 7 = 35, 35 ÷ 7 = 5 and 35 ÷ 5 = 7. Three predictions were generally supported by the results. Firstly, educated adults can factorise two-digit multiplicative products without practice. Secondly, factoring speeds up with practice and transfers to multiplication and thirdly, there is one representation for each triplet that supports both multiplication and factoring. Owing to the reduction in latencies, after practising the problems, it followed that the first part of the model could be cognitively portrayed as: (5, 7, x) ↔ 35, that is, 5 x 7 and 7 x 5 = 35, also 35 can be factorised as 7 and 5 within a single representation; this was the amendment made to form the revised model: the Identical Elements model – revised [IE – r] (*cf.:* Verguts & Fias, 2005 and Butterworth *et al*, 2001).

From the point of view of the present study, the solving of multiple short-division problems may be mediated by multiplication and a set of responses within a cognitive 'multiplication chart' (Campbell, 1997, 1999; LeFevre & Morris, 1999; Rickard, 2005; Rickard & Bourne, Jr., 1996, 1996). Another possibility is that responses may be taken straight from a cognitive 'division chart' within the LTM (Campbell, 1999) or it may be dependent on participant preference (Campbell & Alberts, 2010). There is also the possibility that factorisation may be used, as suggested by Rickard (2005) above, so for example 18 ÷ 3 may be taken straight from LTM, referred to from the point of view of 3 x ? = 18, or 3 is a factor of 18, which factor multiplied by 3 equals 18? As will be seen in Chapter Five the notion that several responses could be activated and the incorrect ones need to be inhibited as suggested by Domahs *et al*, (2007) will be discussed.

2.4 Arithmetic Manipulation, Neuroimaging and Acalculia

Some insight with regard to executive components that might be involved in manipulative procedures such as those described above, carrying, and recognising which arithmetic operation to use is apparent from the literature on acalculic patients; this does not always seem conclusive. Kaufmann (2002) carried out a longitudinal investigation into the mathematical difficulties experienced by a teenager who suffered from severe developmental dyscalculia. MO, a fourteen-year-old male, displayed a discrepancy between relatively normal addition and subtraction fact retrieval but for multiplication and division such fact retrieval was dysfunctional. Although MO's ability to process numbers at a semantic quantitative level was somewhat impaired, his non-verbal abilities such as manipulation of numerical information was sufficiently established to apply self-supportive measures such as finger counting, subitising (recognising small quantities without counting) and magnitude estimation to arithmetic problems. Figure 2.3 is a diagram that displays the interaction in the working memory system that may have an influence on arithmetic fact retrieval; it can be seen that the only hint at any fractionation of the central executive is the 'inhibition mechanisms' box, presumably, in theory, used to inhibit incorrect responses that are close to the intended response.

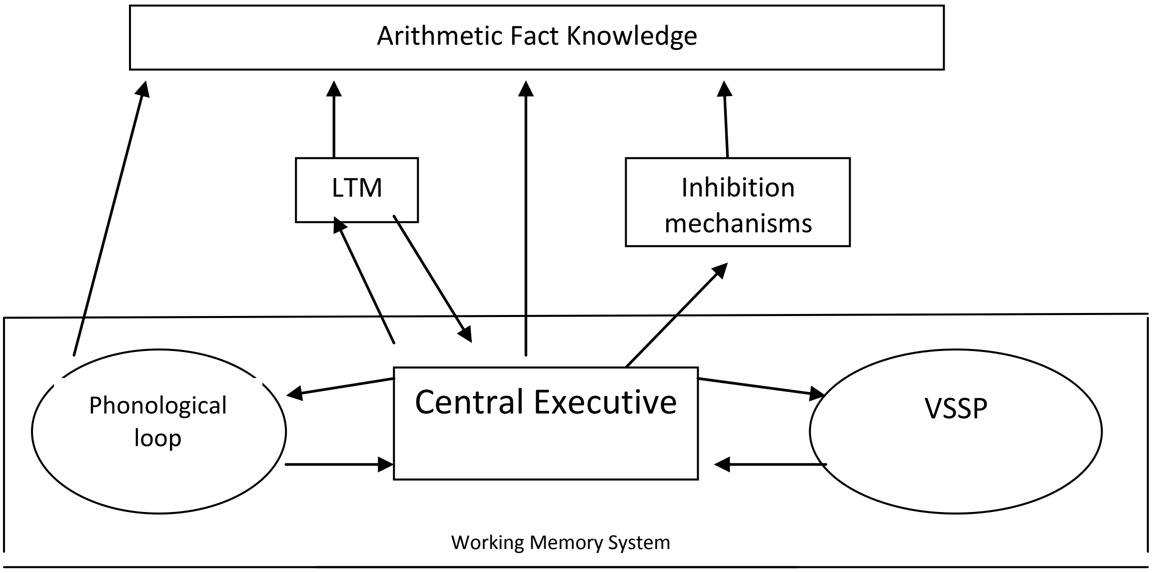

Figure 2.3. A Diagram Representing the Interaction in the Working Memory System Influencing Arithmetic Fact Retrieval. (Adapted from Kaufmann, 2002, p.303).

Kaufmann (2002) concluded that the central executive plays a significant rôle in simple arithmetic although it was also suggested that addition and subtraction may have been carried out by using quantity manipulation (such as decomposing units and decades to make problems more manageable).

Such use of quantity manipulation was noted by Cohen, Dehaene, Chochon, Lehéricy and Naccache (2000) in their case-study of an acalculic patient who had a lesion in her left perisylvian area. Her subtraction and number comparison capabilities were relatively well preserved but her addition and multiplication were impaired in terms of increased error rates.

Moreover, she often resorted to manual representation such as holding up fingers to compensate for her oral output difficulties as a kinaesthetic support mechanism when engaging in addition. The left perisylvian area is thought to be associated with verbal short-term memory (Koenigs, Acheson, Barbey, Solomon, Postle, & Grafman, 2011) suggesting that quantity manipulation may be more of a rôle for the articulatory rehearsal system, with support from the inhibitory mechanism. Kaufmann (2002) suggests that quantity manipulation might bypass these central executive functions, hence the direct arrow from the phonological loop to the arithmetic fact box (see Figure 2.3). However, it was also pointed out that MO's errors may have been the result insufficient access to magnitude representations - one might speculate that these might correspond to the models suggested by Butterworth *et al* (2001) and Verguts and Fias (2005). Kaufmann (2002) also suggested that MO's error proneness could be caused by the faulty application of methods such as finger counting and counting on. Moreover, MO had difficulty performing mental carry operations, pointing to a deficient central executive, as did his very poor fact retrieval skills (Kaufmann, 2002). As will be seen later, this is consistent with Imbo, Vandierendonck, & Vergauwe (2007) who suggested that inhibition was involved in the keeping track of carrying processes in complex addition; furthermore, it is also consistent with the notion that the central executive link to the LTM has connections with arithmetic fact retrieval (Deschuyteneer *et al*, 2006; Imbo & Vandierendonck, 2007).

A study of a young adult patient with developmental dyscalculia and dyslexia was carried out by Kaufmann, Lochy, Drexler & Semenza (2004) as a follow up to a previous case study (Kaufmann, 2002) pertaining to the same patient when he was a teenager. The patient, MO, was compared to a large control group in terms of performance in a number of arithmetic production and verification tasks (Kaufmann *et al*, 2004). The results revealed a much lower accuracy rate for MO in comparison with controls and significantly longer RTs during verification. When verifying addition facts, MO produced the longest latencies for the problems with correct answers. With regard to executive components, the article seemed to be hinting at a significant rôle for input monitoring, long-term memory updating and the phonological loop (particularly the articulatory rehearsal system) when processing tasks that require addition, multiplication and number recognition.

Fulbright *et al* (2000) examined cerebral activation during multiplication of four single digit or low value double digit numbers, also when matching a target number to three previously displayed numbers. There was more activity in the right frontal lobe, an area connected with executive abilities, when participants were engaged in the matching task, and also other areas akin to visual recognition. Multiplication caused more activation in the left frontal lobe region. A later study (Fulbright *et al*, 2003) compared brain activity with regard to geometric shape, letter and *number processing*, in terms of magnitude and spacing. When the numbers were closer together, in comparison with those further apart, the inferior frontal gyri together with the left supermarginal gyrus were activated. There is neuroimaging evidence that the left inferior frontal gyrus is crucial for prepotent response inhibition within the context of Go/No go letter recognition tasks (Swick, Ashley & Turken, 2008). This is consistent with suggestions that closely related responses taken from a semantic number field need to be inhibited if the correct response is to be successfully enacted (see Verguts & Fias, 2005). These suggestions are bound to have an impact on arithmetic problem solving.

In summary, owing to the selective impairments displayed by patients who have suffered lesions to areas of the brain associated with executive functions, it is plausible that different cognitive sub-processes are involved in the comprehension of numerical problems. These brain areas are also associated with calculation procedures, whether these involve retrieval from the LTM, retrieval of rules (*e.g.,* for problems involving ties, zeros and ones as opposed to those containing the digits 2 to 9), carrying, borrowing, approximation, magnitude comparison and processing of operation symbols, to name just some, and also number production (Dehaene, 1992; McCloskey, 1992; McCloskey *et al*, 1985; Warrington, 1982). It is also notable that hints were made with regard to the activation of the articulatory system with assistance from *inhibition* (Cohen *et al,* 2000) when manipulating quantities. Also Kaufman *et al,* (2004) suggests the involvement of input monitoring and memory updating as well as the phonological loop. The purpose of the present study will be to attempt to separate some of these cognitive and arithmetic processes in order to study the relationship between these processes, namely inhibition and division, using a behavioural paradigm.

2.5 Working Memory

Before reviewing the concept of the central executive (inhibition being a subcomponent if this), a little more detail needs to be exposed with regard to different ideologies of working memory. A number of models of working memory have been formulated, three of which will be described here. Demetriou’s model consists of three intertwined systems (see Figure 2.4). The *specialised control systems* depend on the structure of the environment being processed. Once formulated into operative material through the *processing system*, they become knowledge, via the *hypercognitive system*; for example, times-tables might take the form of a list of number facts (of a quantitative structure) that are processed into visual or phonological storage systems ready to be selected, if needed, to solve a problem in the hypercognitive system, either to help solve an immediate problem (for working purposes) or to be retained, long term. The processing system would not function if it were not fed by the specialised control system, in terms of, for example, categorical, quantitative or spatial representation. Speed is indicative of the time required for a mental process to take place, efficiently. Moreover, it would lack direction if it were not controlled by the hypercognitive system. This model has been developed for studying 'theory of mind' concepts in relation to understanding one's own thinking and problem-solving processes (Demetriou *et al*, 2002). It has been shown to provide a basis for studying childhood development in mathematical skills via structural equation modelling (Panaoura, 2007).

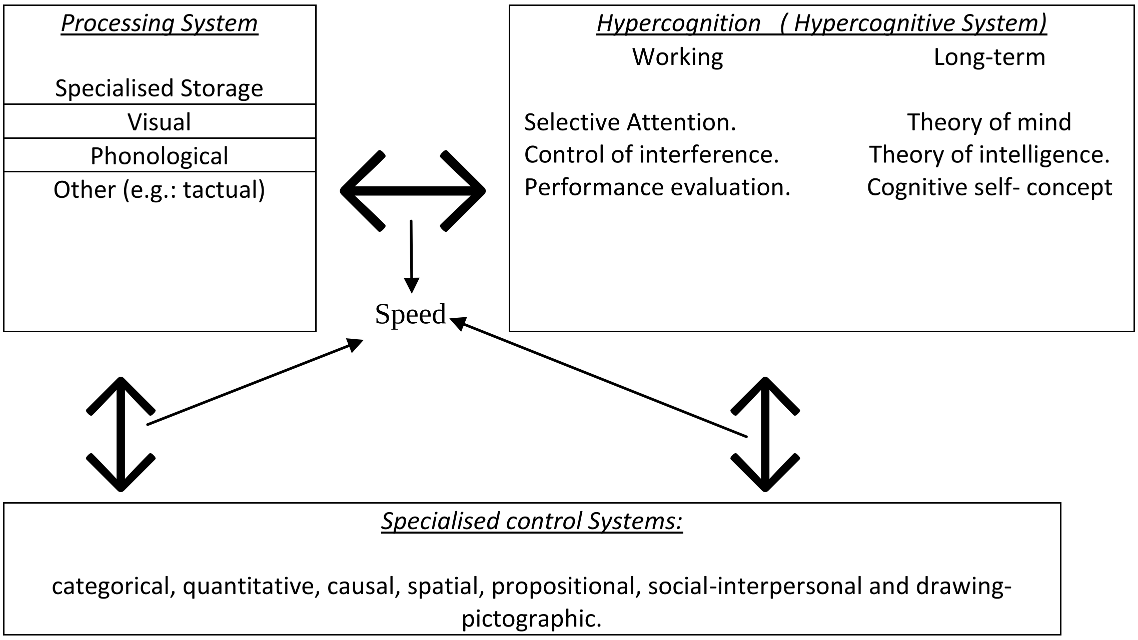

Figure 2.4. Demetriou's Model of Working Memory. (Adapted from Demetriou *et al*, 2002., p. 8)

Cowan (1988) developed a model based on either a parallel or a cascading flow of information processing (see figure 2.5 for a simplified diagram). In this, the short-term memory store is an activated subset of the long-term memory. Stimuli enter the brief sensory store which preserves the physical properties of the stimuli and is thought to be active for a few hundred milliseconds. Information in the long-term store has begun to activate. Habitual stimuli remain in the short-term store but outside the subject's awareness whereas significant stimuli enter the focus of attention. The central executive directs voluntary attention and allows thinking by activating some information in the long-term store. Activation of pre-motor and motor pathways in the short-term store results in actions. The original diagram was positioned above a horizontal axis representing an ordinal timeline, presumably to emphasise the temporal flexibility of information processing (Cowan, 1988). In brief, this model is based on short term storage linked to activated-long-term memory and an attention controlling central executive system with a capacity of four chunks (Cowan, 2001).

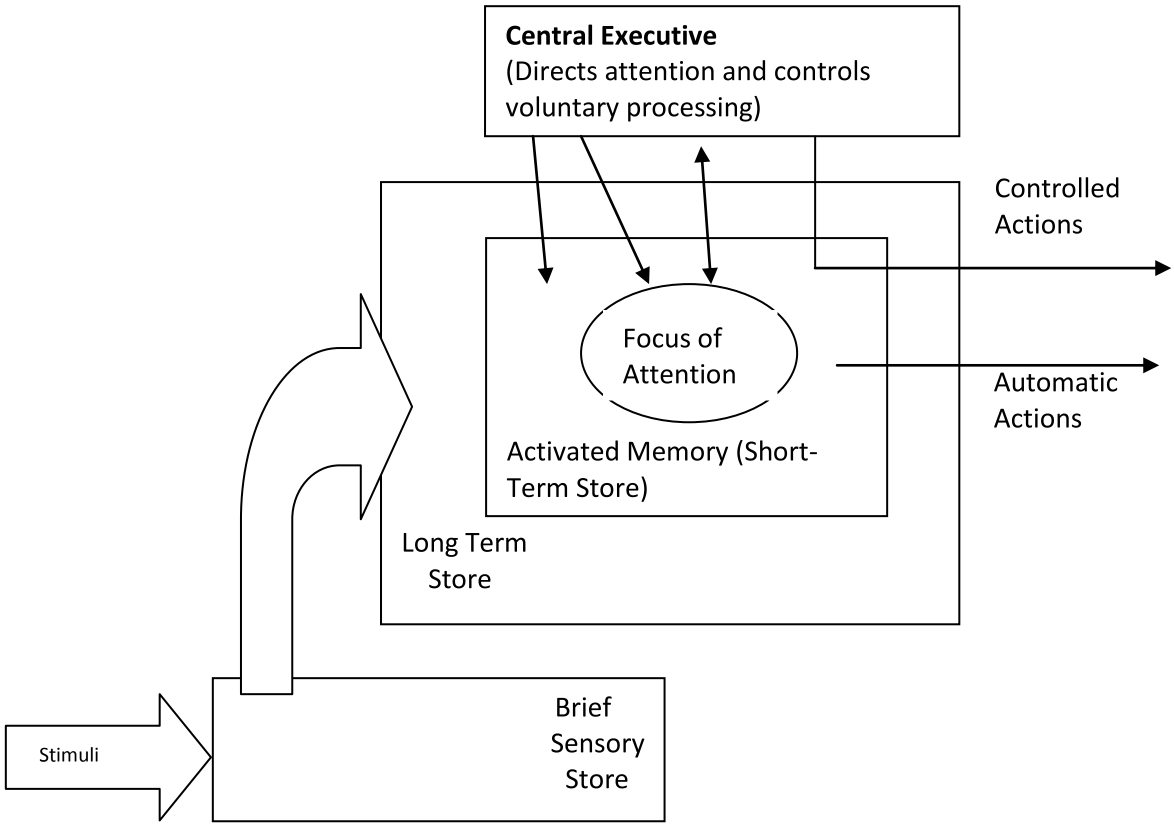

Figure 2.5. A Simplified Version of Cowan's Model of Information Processing (Cowan, 1988; adapted from p. 180)

Illustrated in Figure 2.6 is a diagram of the multi-component model of working memory proposed by Baddeley (1997) and how it was later updated to include the *episodic buffer* (Baddeley, 2000). The working memory model originally proposed by Baddeley & Hitch (1974) consists of three components: the visuo-spatial sketchpad, the central executive and the phonological loop. In performing some kind of mental task, the phonological loop acts as a temporary storage system for verbal material and is also involved in recycling phonological information for immediate recall. For example, as will be seen later, Fürst and Hitch (2000) suggested that, when carrying is required in a multi-digit addition problem, the phonological loop may serve as storage device for intermediate results. The visuo-spatial sketchpad processes visual imagery tasks, spatial and visual search assignments and acts as a temporary storage system for visuo-spatial material. The rôle of the central executive is in decision making, reasoning, language comprehension, allocating the relevant proportion of visual or spatial resources needed to process a particular task and to transfer information to the long-term memory via rehearsing and recoding (Ashcraft, 1994; Baddeley, 1997; Logie, 1995). This was later updated to include the Episodic Buffer, the rôle of which is to take any necessary information from the long term memory that is required for the successful execution of the task in question (Baddeley, 2000).

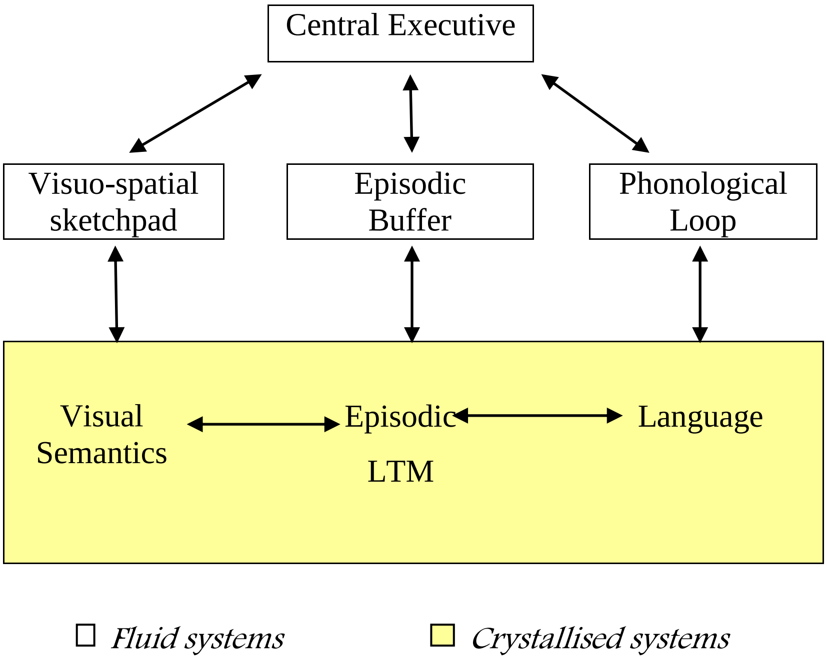

Figure 2.6. The Current Multi-Component Model of Working Memory. Adapted from Baddeley (2000) p. 421.

The white areas are the fluid systems with a limited capacity; a limited capacity in that they are used as a temporary store for information whilst a specific problem is being solved. The phonological loop can be fractionated into the articulatory rehearsal system for rehearsal of, for example, intermediate results in complex arithmetic, and a temporary storage container, for example, an arithmetic problem in the process of being solved. The visuo-spatial sketchpad is for manipulating visual material. The episodic buffer is assumed to feed episodic information into the long term memory and also to receive such information from it. It forms a temporary integrational interface between the visuo-spatial and phonological slave-systems. Mechanisms for modelling the environment, creating new cognitive representations and the facilitation of problem solving are thought to be contained within the episodic buffer (Baddeley, 2000, 2003). The rôle of the central executive is in decision making, reasoning, language comprehension, allocating the relevant proportion of visual or spatial resources needed to process a particular task and to transfer information to the long-term memory via rehearsing and recoding (Ashcraft, 1994; Baddeley, 1997; Logie, 1995).

Although Demetriou's model is a useful basis for studying developmental processing efficiency in relation to self awareness and self-concept (Demetriou *et al*, 2002; Panaoura, 2007), its components are strongly intertwined, rendering it less practicable for investigations of a behavioural nature. It is notable that the executive functions, interference control, selective attention and performance evaluation form part of the hypercognitive system. This leaves two multi-component models. Cowan's model devotes more explanation to the processing of background stimuli as well as that of primary interest to the subject. The more practicable model, with components and sub-components that are separable, is Baddeley's. Baddeley's multi-component view of working memory has been regarded as a useful method of studying the cognitive mechanisms on which arithmetic processing relies (Logie, Gilhooly & Wynn, 1994). Moreover, as can be seen from Figure 3, it has separate visuo-spatial and phonological systems and, as a model it has helped and continues to help guide research into specific cognitive processes (Andrade, 2001). Although all three models lend themselves to factor analytical studies owing to the similarities of common components such as the central executive and components that deal with sensory attributes of stimuli, it does not follow that all three models lend themselves to a behavioural paradigm. For these reasons, the research referred to in the present study has been entirely based on Baddeley's (2000) model in order to maintain consistency with other behavioural researchers' methodology. The main attentional control component of this model is the central executive

2.6 The Central Executive

The Central Executive has been regarded as the most important subsystem of working memory since Baddeley and Hitch (1974) first proposed their (three) component model owing to its capacity to be able to coordinate visual and phonological resources to specific tasks; other capacities have since been proposed, as will come to light, later. 'Three' is in brackets in the previous sentence because, at the time, it appeared somewhat tentative that it was a single entity, namely a unitary control system that dictated the magnitude of cognitive resources to be distributed between the phonological and visuo-spatial stores. There existed the slight possibility of a separate central executive for each of the phonological loop and the visuo-spatial sketchpad (Baddeley & Hitch, 1974). Further research and synthesis of findings, particularly when taking into consideration studies of patients with frontal lobe damage and ageing persons with dementia, highlighted the strong possibility of a single central executive: attentional processing and long-term memory linkages being at least some of its functional rôle (Baddeley, 1986). Some twenty years after Baddeley's and Hitch's earlier work (1974), a single central executive had been accepted but it was suggested that this component could be fractionated into separate subcomponents (Baddeley, 1996).

2.7 The Fractionation of the Central Executive

The concept of the central executive has not been without controversy. Kimberg & Farah (1993) and Parkin (1998) argued against the existence of the central executive on the grounds that the functions of this component could not be a single entity, nor be attributed to a single area of the brain, namely, the frontal lobe region. In the light of a computer simulation of the behaviour of patients with frontal lobe lesions, it was proposed that frontal lobe damage affected processing components of tasks rather than executive components for their co-ordination (Kimberg & Farah, 1993). Baddeley (1998) found it necessary to refute Parkin's argument by maintaining that central executive functions could not be mapped on to a single area of the brain; he was adamant that central executive processes most likely involved links to different parts of the brain. Earlier, Baddeley (1996) had proposed four fractionated subcomponents: input and output monitor, to co-ordinate performance on two separate tasks; a response selector, to change retrieval methods and for decision making; a selective stimulus inhibitor, for allowing the processing of only relevant information; and a long-term memory holder and manipulator, for the updating of memory and for retrieval from the long term memory store. Interestingly, a comparatively early neurological study (D'Esposito, Detre, Alsop, Shin, Atlas, & Grossman, 1995) using fMRI demonstrated the employment of the dorsolateral prefrontal cortex when participants performed a dual-task exercise. This area did not operate when the same tasks were carried out as single activities. The central executive subcomponent thought to be responsible for dual task co-ordination is input monitoring (Baddeley, 1996).

Some ten years later, there does not necessarily appear to be complete consistency with regard to the precise number of central executive functions but similarities would appear to be evident between inhibition-types with differing terminologies. For example, Fournier-Vincent, Larigauderie, & Gaonac'h (2008) put forward a very strong case for six processes, namely: (1) verbal storage and processing coordination; (2) visuo-spatial storage and processing information; (3) dual-task coordination; (4) strategic retrieval; (5) selective attention; and (6) shifting. This paper employed structural equation modelling to try to gauge the separabilty of these six executive functions. They were all found to be separable apart from dual-task coordination. The dual-task coordination measures apparently lacked convergent validity and were therefore dismissed from the analysis. It was, however, emphasised that they could not be regarded as completely independent: the processes would need to be permitted to interact, to a greater or lesser extent, depending on the cognitive activity being undertaken (also, *cf.* Miyake, Friedman, Emerson, Witzki, Howerter & Wager, 2000; Friedman & Miyake, 2004). An earlier study by Fournier, Larigauderie, & Gaonac'h (2004) had already examined four executive functions: coordination (input monitoring), inhibition, retrieval from the long-term memory and planning. The additions in the later (2008) article may equate to parts of the central executive that coordinate and allocate visuo-spatial and phonological resources, depending on the task to be undertaken.

Not only can the central executive be fractionated but also can at least some of the executive functions. Head *et al* (2008) studied the volumes of a number of cortical areas in relation to ageing and episodic memory. It was found that the volume of the prefrontal cortex had an effect on episodic memory via working memory and inhibition. Their results supported the notion that inhibition control is linked to memory retrieval and, furthermore, that successful retrieval depends on this inhibitory control. Inhibition supposedly controls the flow of information within working memory and serves not only to prevent the intrusion of irrelevant information but also to suppress information that is no longer relevant. Interestingly, from a neuropsychological point of view, Roberts and Pennington (1996) suggested that inhibition was more than just a requirement for suppressing inappropriate prepotent responses and, as an aspect of terminology, was used too broadly; it needed to be examined and looked upon with greater specificity. Friedman and Miyake (2004) made an in-depth examination of inhibition. Methodologically speaking, they split this function into 'prepotent response inhibition,' 'resistance to proactive inhibition' and 'resistance to distracter interference.' Prepotent response inhibition is defined as the ability to suppress dominant, automatic or prepotent (*i.e.:* more powerful) responses whereas 'resistance to proactive inhibition' is the ability to resist memory intrusions from information that has been relevant but is now irrelevant. Resistance to distracter interference is recognised as the ability to filter interference from the external environment that is not relevant to the task being executed. This is consistent with an earlier study by Barkley (1997) where behavioural inhibition was categorised into prepotent response inhibition, on-going response inhibition and interference control. The main emphasis was that, if inhibition is studied, it needs to be made very clear which inhibition *process* it is (also *cf.:* Bull and Scerif, 2001).

An earlier study by Fournier *et al* (2004) examined four executive functions: coordination (input monitoring), inhibition, retrieval from the long-term memory and planning, three of which were further fractionated. Two distinctions were assumed for coordination: integrating information that is derived from different sources, and the coordination of two unrelated tasks. Inhibition was assumed to have three abilities: prepotent response inhibition; the discarding of irrelevant information and attending to a relevant stream of information; and inhibiting what is no longer required in favour of attending to new information (*cf.:* Bull and Scerif, 2001) and is consistent with the findings of Friedman and Miyake (2004). Planning was analysed in terms of plan formulation, regulation of responses required to execute the plan and verification that the plan was following the desired pathway.

If the two Fournier(-Vincent) *et al* papers are compared (2004 and 2008), it is of interest that in the 2004 article, four central executive functions were initially present, including planning. Within the 2008 paper, planning is not directly evident as a subcomponent for analysis but two new coordination and storage processes have been added: verbal storage and processing coordination; and visuo-spatial storage and processing of information. As previously explained, dual-task coordination was removed from the latent variable analysis owing to its lack of convergent validity and the resultant suggestion that this process was not completely separable. One of the main findings of the 2008 study was that executive functions are separable but not independent. The final five target functions needed to interact with one another in order to operate efficiently. It was further proposed that the rôle of the central executive, based on the results and conditions of this (2008) paper, was that it exists to permit interaction between selected processes, as required by the particular cognitive activity that is being executed. Interaction between pairs of executive abilities appears has been hinted at by experimental approaches, as will be seen later.

In summary, although the number of executive subcomponents is a matter of a limited amount of controversy, there is consistency with respect to four of them: dual task co-ordination or input monitoring (Baddeley, 1996; Fournier *et al*, 2004; Fournier-Vincent *et al*, 2008); retrieval form LTM or memory updating; planning or response selection (Baddeley, 1996; Fournier *et al*,2004) and inhibition (Baddeley, 1996 ; Fournier *et al*, 2004) or selective attention (Fournier-Vincent *et al*, 2008). Considerable consistency is evident across the literature with regard to the separabilty of inhibition into prepotent response inhibition, resistance to distracter interference, and resistance to proactive interference (Barkley, 1997; Friedman & Miyake, 2004; Fournier *et al*, 2004; Head *et al*, 2008). The main focus of the present study was prepotent response inhibition and resistance to distracter interference; as will be seen later, suitable activities were created to load these subcomponents so they could be studied within a behavioural paradigm.

2.8 Correlational Approaches Comparing Arithmetic Performance with that for Executive Abilities

One method of studying the central executive's rôle in arithmetic is by testing executive functions in children and carrying out a correlational comparison between these test results and scores on arithmetic assessments (Andersson, 2008; Bull & Scerif, 2001; Espy, McDiarmid, Cwik, Stalets, Hamby, & Senn, 2004; Gathercole & Pickering, 2000; Keeler & Swanson, 2001). The results from separate tests for executive-specific processes have been compared with those of arithmetic assessments.

Before reviewing some correlational studies, a different type of comparative approach has been to take two samples of participants and carry out a between-subjects exercise involving one group classified as having low arithmetical achievement being compared to an average achieving control group, in terms of scores on both central executive and arithmetic assessments. D'Amico and Guarnera (2005) carried out such a study on a sample of children in order to explore the relationships between working memory and arithmetic achievement. With respect to central executive functions, it was evident that, as highlighted by this type of analysis, there were significant rôles in arithmetic processing, from a developmental point of view, for input monitoring (dual-task coordination), switching (response selection), and memory updating, as executive components. There was also evidence of involvement of the slave systems, namely, the articulatory rehearsal system portion of the phonological loop and the visuo-spatial sketchpad (D'Amico & Guarnera, 2005). Of most interest with regard to the present study are the rôles of executive components.

Andersson (2008) matched a number of activities with the central executive function it was thought to test and correlations were calculated between these and arithmetical tests consisting of the arithmetic calculation standard (e.g., 343 + 96 and 836 – 248), arithmetic equations (e.g., 63 + _ = 94 and _ x 4 = 16), arithmetic combinations (e.g., 6, 17 = 23 and 10, 50, 90, = 30 ↔ put in the operation +, -, or x) and arithmetic fact retrieval. From the results, it was indicated that input monitoring, for the coordination of multiple processes during calculations, was a crucial executive function. Also crucial was the long-term memory link to access number facts and procedural schemata from the LTM. It was also emphasised that response selection was important for switching or shifting between internalised cognitive groups of procedural and arithmetical knowledge. This is at least partially consistent with Bull and Scerif (2001) who carried out a similar investigation, although with children who were some 3 years younger (M = 7 years 4 months, range: 6 years 9 months to 8 years 3 months) placed in Primary 3, in Scotland. They had already suggested that children with a lower mathematical ability probably found it difficult to inhibit prepotent information and previously learned methodologies. However, they found no significant correlation between mathematical ability and input monitoring; this was consistent with the lack of involvement of input monitoring found by Deschuyteneer & Vandierendonck (2005a, 2005b) in simple addition and multiplication. It was of particular note, in relation to the present study, that inhibition was effectively fractionated into inhibiting (1) prepotent information, (2) previously learned methodology, and (3) information previously held in the long-term memory. The first and third types of inhibition are similar to those confirmed by Friedman & Miyake (2004).

A wider viewpoint was presented by Gathercole and Pickering (2000) who carried out an inter-correlational analysis of working memory tasks and Key-Stage-One standard assessment tasks (SATs), hence English and science were examined in addition to mathematics. The results suggested that the central executive plays a crucial rôle in children's acquisition of complex literacy, comprehension and arithmetic skills. More specifically, they referred to the processing of new information and synthesising it with concepts held in the long-term memory, hence hinting at the long-term memory holder and manipulator, proposed by Baddeley (1996). No specific rôle was found for the phonological loop, in contrast to Bull and Scerif (2001).

Espy *et al* (2004) studied executive functions in 2 to 5 year old children and found that inhibition played a very important part in emergent arithmetic proficiency. Keeler and Swanson (2001) took an overall different approach and compared children's declarative knowledge of arithmetic methods rather than their procedural knowledge with working memory capacity. In other words they examined the children's knowledge with regard to their use of the most appropriate algorithm or arithmetic procedure for a particular problem. One conclusion was that when the storage capacity of working memory is exceeded, some form of executive system, it was assumed, accesses the long-term memory in order to retrieve procedural knowledge (Keeler and Swanson, 2001). This seems feasible; however, if the knowledge from LTM is not available – probably because it had never been learned or absorbed - then rehearsal mechanisms or some other executive function may need to be employed by the cognitive system.

There is evidence that people who suffer from specific anxiety when asked to solve mathematical problems have a reduced working memory capacity, from a procedural perspective. Ashcraft and Kirk (2001) carried out a correlational study on adults. They found that people with high mathematics anxiety had particular difficulty with carrying procedures when pursuing double column addition, suggesting that less working memory capacity was available to devote to such procedures under a memory load condition. It was concluded that high mathematics anxiety resulted in a reduction of working memory capacity when procedures such as carrying, borrowing, holding interim results and sequencing are required. A further suggestion was forwarded that the locus of this effect was in the central executive; specific central executive functions were not postulated but it was proposed that future research might examine the working memory's specific rôle in mental arithmetic, with particular respect to the implementation of arithmetic procedures. The possibility was also raised that negative anxiety-related thoughts, being attention-related, might present themselves in the central executive and interfere with arithmetic procedures. Amongst the aims of the present study will be to study the 'potentially difficult' carrying procedures under the load of prepotent response inhibition and resistance to distracter interference.

In summary it has emerged from the correlational literature that, from a developmental perspective, the central executive is involved in processing of new information and synthesising it with arithmetic concepts present in the LTM (Gathercole & Pickering, 2000). More specifically input monitoring, response selection and memory updating are involved in children's arithmetic processing (D'Amico & Guarnera, 2005), particularly input monitoring for co-ordinating arithmetic processes, memory updating for accessing number facts and response selection for selecting appropriate procedural and arithmetic knowledge from LTM (Andersson, 2008). Moreover, inhibition plays a part in learning new arithmetic skills (Espy *et al,* 2004) and, as new skills are learned, inhibition of prepotent information and of previously learned arithmetic methodology become increasingly important (Bull & Scerif, 2001).

Correlational approaches can be useful for predicting cognitive functions (generally speaking) such as mathematical ability by comparing scores on tests of executive functions with those of arithmetic competence. Experimental methods, on the other hand, can be more direct with respect to matching working memory functions to what might be termed behavioural functions such as reading processes and mathematical procedures (Baddeley, 2007) [but researchers appear to be a little more tentative with regard to the accuracy of match between suppression activities and the processes they are deemed to load]. Baddeley appears to be hinting that what is required is a methodology to map cognitive functions such as arithmetic processes onto the underlying and readily modelled working memory components and subcomponents. To this end, it may be the case that experimental rather than correlational methodology and regression analyses are likely to yield greater reliability (Baddeley, 2007). The present study was intended to be a purely behavioural study and before delving into the aims of the present study, a wealth of literature exists that has examined the executive component (and its subcomponents) within a behavioural paradigm.

2.9 Behavioural Approaches to Studying the Rôle of the Central Executive in Mental Arithmetic

It has been seen that some engaging conclusions, with regard to the rôle of the central executive and its subcomponents, are evident in the literature on correlational studies. What one might term experimental or behavioural approaches directly used the dual-task paradigm (Deschuyteneer & Vandierendonck, 2005a, 2005b; Deschuyteneer, Vandierendonck,&

Muyllaert, 2006; Duverne, Lemaire, & Vandierendonck, 2008; Imbo & Vandierendonck, 2007; Imbo, Vandierendonck & De Rammelaere, 2007; see also: Imbo, De Rammelaere & Vandierendonck, 2005; Logie, Gilhooly & Wynn, 1994). A relatively early account (Logie, Gilhooly, & Wynn, 1994) examined the central executive and the slave systems in relation to two-digit addition and found that working memory played a crucial rôle. More specifically, with regard to the central executive, which was loaded by the implementation of random letter generation, it was concluded that this played a major part in the calculation procedures and for producing approximations of the correct answers. De Rammelaere, Stuyven & Vandierendonck (2001) investigated whether or not the central executive and the phonological loop were involved in simple sums and products. There were problems, at the time, developing suitable suppression techniques for the separate fractionated functions of the central executive. They instructed their participants to tap an unpredictable rhythm on the zero key of the numeric keypad whilst answering the problems. The phonological loop was found not to be involved but the central executive was, in that its suppression significantly increased the latencies. It was not clear which particular executive function this was, however, it was speculated that verification of the correct answer might be its involvement. This seems plausible enough as the participants were asked to state their answers to verification tasks and might suggest response selection.

Somewhat later, more specific suppression techniques were being developed aimed at separate executive functions. Deschuyteneer and Vandierendonck (2005a) examined the rôle of input monitoring and response selection when solving simple addition problems (e.g., 7 + 6). They used the dual task paradigm to suppress these two executive processes. Participants were instructed to press the [0] key on the numerical keypad whenever they heard a tone (262Hz) which was presented every 1700ms whilst carrying out the arithmetic task. For a second condition the tone was sounded randomly, at intervals of 900 and 1500ms. The results of the two conditions were compared. They concluded that response selection was involved in simple addition but input monitoring was not. They did, however, question whether the secondary task was taxing enough in terms of variety of lengths of the intervals between the two tones and suggested future research might use a greater number of temporal intervals. Similarly, Deschuyteneer and Vandierendonck (2005b) used much the same method to study the rôle of input monitoring and response selection for simple multiplication. Their results suggested a significant purpose for response selection, very likely for retrieving arithmetic facts from the long-term memory but no significant rôle for input monitoring. Deschuyteneer *et al* (2006) studied simple sums and products under both response selection and memory updating loads. Results of the RT analysis suggested a significant effect of both response selection and memory updating on simple addition. Some consistency was therefore evident regarding the rôle of response selection. Similar findings were reported by Duverne, *et al* (2008).

This suggestion is comparable with the notion forwarded by De Rammelaere, Stuyven, & Vandierendonck (2001) that the CE is involved in verification of the correct response. Assuming that verification is part of the *response selection* process, this seems plausible. Accuracy rates were too high to spotlight any significant differences between loaded and control conditions. Correlations were calculated between RTs in both arithmetic and secondary tasks; as they were not significant, no trade-off between performances was suggested within the concurrent primary and secondary activities. With regard to multiplication, significant effects were indicated by both response selection and memory updating loads, however, the effect of memory updating was significantly larger than that for response selection. They did, nevertheless, question the use of the 'one-back two-choice' reaction time task for loading memory updating where participants responded to two tones (262 and 524Hz) each lasting 200ms presented 1700ms apart. Participants pressed the key corresponding to the *penultimate* tone heard ([1] – 262Hz; [4] – 524Hz). They raised the possibility that this might also have hindered other executive functions such as interference control (Deschuyteneer *et al,* 2006). The interference control referred to, here, is probably what Friedman & Miyake (2004) referred to as resistance to proactive interference.

Simple multiplication and division were examined by Imbo and Vandierendonck (2007) under central executive load. It was suggested that central executive resources were employed for the retrieval of multiplication and division facts from LTM, and also whenever non-retrieval methodologies were used. The central executive did not appear to play a rôle in strategy selection. The Phonological loop was involved in the storage of intermediate and partial results. Consistent with these proposals, Baddeley and his colleagues emphasised the need for executive processes to be controlled by verbal processes, effectively suggesting a significant supporting rôle for the phonological loop for assisting switching or response selection (Baddeley, Chincotta & Adlam, 2001). They carried out a series of seven experiments examining the effects of phonological and executive suppression on switching between addition and subtraction (single digit numbers + 1 or – 1). However, the signs were not always present. Overall, it was concluded that the central executive was involved in switching and the phonological loop had an essential rehearsal rôle when the signs were absent. Moreover, verbal control (phonological loop support) of executive processes is an important function (Baddeley *et al,* 2001). As the tasks involved alternating between two arithmetic operations, the theoretical implications here, are that response selection was involved in selecting the required arithmetic operation, furthermore, if the signs were missing, it was all the more important for memory to be assisted by the phonological loop.

The studies reported on so far used simple arithmetic as the target material; the reference to 'simple' denotes problems involving single-digit numbers. From a cognitive perspective simple arithmetic, certainly in adults, mainly involves the retrieval of responses from the long term memory but may involve some counting on. The executive subcomponents for these, it was proposed, are likely to be a combination of memory updating and response selection (Deschuyteneer *et al*, 2006). More complex arithmetic is likely to be more proceduralised and involve multiple arithmetic operations, decomposition of larger numbers and carrying or borrowing, taking a far greater number of cognitive resources. Imbo, Vandierendonck and De **Imbo,** Vandierendonck, & Rammelaere (2007) examined the rôle of working memory in more complex arithmetic, involving carrying. Not only did they investigate the effects of the number of carry operations and the value of the carries in complex addition, they also looked at the respective rôles of the phonological loop and the central executive. According to the secondary tasks employed, two central executive processes: inhibition (prepotent response inhibition) and response selection were effectively studied. The central executive loads were found to have a significantly detrimental effect on accuracy rates, particularly that for inhibition, when processing carrying operations. Related to this, **Imbo,** Vandierendonck, & Vergauwe (2007) experimented with complex subtraction (2-digit- 2 digit and 4-digit- 4digit) and complex multiplication (2-digit x 1-digit and 3-digit x 1 digit) and examined both the phonological loop and the central executive. It was found that as the number of carry operations increased, executive load was elevated. This was interpreted in terms of the strong tendency not to carry having to be inhibited. Similar conclusions were drawn by Fürst & Hitch (2000). Furthermore when solving complex problems that involve carrying, conflict arises between the carry and no-carry tendencies which need to be resolved. More executive resources are also needed to keep track of the carry operations as they increase (**Imbo,** Vandierendonck, & Vergauwe 2007).

Thus far, the experimental approach has been overviewed where the dual-task paradigm has been used to compare latencies and error rates under concurrent performance of arithmetic and executive-specific and, where appropriate, slave-system-specific activities; the results being analysed by general linear model analyses of variance. In summary, it has so far been suggested that simple sums and products use central executive resources (De Rammelaere *et al,* 2001), more specifically, response selection (Deschuyteneer & Vandierendonck, 2005). Simple products, consistent with Deschuyteneer & Vandierendonck (2005), rely on response selection but not on input monitoring (Deschuyteneer *et al*, 2006). Duverne *et al* (2008) suggest that the central executive is responsible for the retrieval of multiplication and division facts from the LTM and response and strategy selection, although this slows with age. Inhibition and input monitoring are both involved in carrying operations when complex addition is executed (Imbo, Vandierendonck & De Rammelaere, 2007). The overall aim of the present study is that the behavioural paradigm will be taken a step further and suppression activities created that specifically load prepotent response inhibition and resistance to distracter interference in order to study their respective rôles in complex division.

2.10 The Purpose of the Present Study

It has been seen that much research has been carried out into number recognition, and addition, subtraction and multiplication, both simple and complex. Although a good deal of research has been carried out into number recognition (e.g., Dehaene, 1992; McCloskey, 1992), more relevant to the present study is the three-stage model proposed by McCloskey *et al* (1985) containing comprehension, calculation and production. It was envisaged, for the purpose of the present study, that the last two elements may be where inhibition might be enacted: during calculation, to inhibit irrelevant methodologies; and production, to inhibit conflicting responses. Warrington (1982) suggested that approximation, exact number fact retrieval and arithmetic processing were separable which raised the question as to whether or not partial calculations and carrying, as procedures of a complete problem are separable: such an attempt will be made, in the present study in order to verify or otherwise any phenomena that emerge in the first two experiments.

Turning to the arithmetic operation to be studied, the semantic field proposed by Butterworth *et al* (2001) for addition and that for multiplication (Verguts & Fias, 2005) may be regarded as part of either the calculation process or the production process depending on individual differences. There is evidence that suggests different people use different arithmetic methodologies, for example, when solving a division problem, responses may be taken straight from a cognitive 'division chart' within the LTM (Campbell, 1999) or it may be dependent on participant preference (Campbell & Alberts, 2010). Furthermore, in the case of division, responses may be mediated by multiplication (Campbell, 1997, 1999; Rickard, 2005; Rickard & Bourne, Jr., 1996, 1996). It has also been evident that there are a multitude of methodologies for solving arithmetic problems (Dehaene & Cohen, 1997; Heirdsfield, 2002; Heirdsfield & Cooper, 1997; LeFevre *et al*, 2003; LeFevre *et al*, 2006; Verguts & Fias, 2005). Furthermore, the researchers previously referred to carried out their research on addition, subtraction and multiplication. Little has been written about complex division, the present study will go some way to filling this gap in the literature.

To reiterate, the cognitive processes involved in arithmetic division, as will be expanded upon in the introduction to Chapter Three, have received little attention in comparison to addition, subtraction and multiplication. The present study investigated the effects of prepotent response inhibition load and resistance to distracter interference on division problems comprising a four-digit dividend (always beginning with 1) and single-digit divisors ranging from 2 to 9, inclusive.

Owing to the lack of literature pertaining to division, the possible methodology used by adults was taken from the Author's past experience. Certainly, with regard to complex division, those who were at school during the latter half of the twentieth century, in England and Wales, would have been taught a left to right method which may or may not have involved the carrying of remainders, depending on the complexity of the individual division problem. Further explanation of the methodology is in the introduction to Chapter Three; it encouraged the use of the left-to-right process as described above but using the format:

$$\frac{1425}{5}$$

The reason for using this particular top-heavy fraction format lies in the ease of integrating the flanker digits, for example,

$$\begin{array}{c} 777\underline{1425}777 \\ 777\underline{5}777 \end{array}$$

It would not have been as practicable to integrate flanker digits if the problems were in the format 1425/5 or 1425 ÷ 5. From the point of view of executive abilities, the main focus, at least in the first part of this study, is on two central executive abilities, namely prepotent response inhibition (PRI) and resistance to distracter interference (RDI) and how these relate to complex division. These two subcomponents were focussed upon in order to complement other literature on the subject of central executive subcomponents and mental arithmetic. Other executive subcomponents such as response selection, input monitoring and memory updating have received attention, within the context of mental arithmetic (Deschuyteneer & Vandierendonck, 2005a, 2005b; Deschuyteneer *et al*, 2006).

A major consideration was to decide upon suitable secondary tasks to load the two executive abilities in question. Friedman & Miyake (2004) assessed prepotent response inhibition with a version of the stop-signal task. Early studies involving this were designed to examine the phenomenon that a primary task can be inhibited if it is closely followed by a second stimulus (Helson & Steger, 1962; Lappin & Eriksen, 1966). Carter *et al* (2003) developed their own version where participants were instructed to respond to the direction in which pictures of an aeroplane were pointing (left or right); after a Stop-Man was displayed, they were to stop responding until a green 'go-signal' was shown. In the present study this was not deemed practicable to use as a secondary task concurrently with solving complex division problems; there would have been too much information to absorb. It was decided to use a dual task to build a prepotent state of stating the direction of arrows with the division problems superimposed on them. Following a red screen instructing participants to *stop* saying, “left / right,” there followed a new condition requiring participants to only respond to the division problems; the arrows were, however still present. The logic was that participants now had to *inhibit* saying left or right in response to the arrows. Full methodological details are in Chapter Three. Strictly speaking, this was not a dual task activity but a dual task activity with a priming purpose followed by the required experimental condition.

For RDI, it was decided that the most appropriate task was an adapted version of the Eriksen Flanker Task (Eriksen & Eriksen, 1974). This, along with the Stroop Task (Stroop, 1935) is thought a suitable activity to assess the resistance to distracter information system (Friedman & Miyake, 2004). It was not thought that the Stroop task would be suitable as an adaption owing to the visual complexity of the division problems. However, the top-heavy fraction format of the division problems made the Eriksen Flanker task an ideal adaption because the flanker digits could easily be slotted in on either side of the dividend and divisor. It had already been employed by other researchers to study magnitude verification and the effects of numerical distance between the flankers and the target digit (Censabella & Noël, 2005; Notebaert & Verguts, 2006; Nuerk, Bauer, Krummenacher, Heller & Willmes, 2005; Ullsperger, Bylsma & Botvinick, 2005); this is explained further in Chapter Four. The present study represents an extension of the use of the Eriksen Flanker Task for investigating more complex numerical processing.

To summarise, Experiment One focusses on prepotent response inhibition (PRI) and was designed to investigate whether or not PRI has any involvement in the solving of a complete division problem. The second Experiment was designed to ascertain whether there is any rôle for resistance to distracter interference in the solving of complete division problems. Realising that complex division is a multi-proceduralised process, Experiment Three was designed to attempt to extract one of these procedures, namely that of short division and determine the responsibility, if any, of both PRI and RDI. The final experiment represents an attempt to extract the carrying procedure and perform similar investigations as those carried out in Experiment Three. The original notion was that any effects in the first two experiments may be at least partially caused by either PRI or RDI (or both) taking responsibility for one or both of the two arithmetic procedures studied. The findings, with respect to this notion, are presented in the General Discussion.

CHAPTER THREE

The Effect of Prepotent Response Inhibition on Mental Arithmetic Division

Since it was proposed by Baddeley (1996) that the central executive component of working memory could be fractionated into four sub-components, it has been later proposed that further separabilty may be evident with respect to the subcomponents. Working memory can be regarded, in its most basic form, as a four component cognitive processing system consisting of a visuo-spatial sketchpad for dealing with visual and spatial information, a phonological loop for processing phonological information and an interface for synthesising both types of information and combining it with, where necessary, long-term memory storage and retrieval. These three components are co-ordinated by the central executive component (Baddeley, 2000). The four sub-components of the central executive can be briefly described as follows: an input monitor for co-ordinating two separate tasks; a device to change retrieval strategies, for response selection; a holder and manipulator of long-term-memory information, for memory updating; and a selective stimulus inhibitor, described in more detail later (Baddeley, 1996). The focus of the present experiments is an attempt to examine one of these sub-components, namely the stimulus inhibitor, while keeping input monitoring, response selection and memory updating in the immediate background (*cf*. Baddeley, 1996, 1998; Parkin, 1998). Inhibition is very likely a mechanism that controls the flow of information through the working memory system (Head, Rodrigue, Kennedy & Raz, 2008). Recent neurological studies have suggested that the term inhibition has been used too broadly in that it was regarded primarily as a mechanism for inhibiting unwanted informational intrusions and was really a general term covering, more specifically: pre-potent response inhibition, inhibition of previously required information that is no longer required – resistance to proactive interference, and resistance to distracter interference (Barkley, 1997; Friedman & Miyake, 2004; Roberts & Pennington, 1996). The more specific focus of the present experiment is intended to be on prepotent response inhibition in order to investigate its rôle, if any, with regard to the procedures involved in solving complex division problems. Prepotent response inhibition is defined as the ability to suppress dominant or automatic responses (Friedman & Miyake, 2004).

Previous studies examining the function of executive abilities within the context of arithmetic processing have focussed on the central executive without necessarily specifying a particular fractionated executive function (e.g., De Rammelaere, Stuyven & Vandierendonck, 2001; Fürst & Hitch, 2000; Lemaire, Abdi & Fayol, 1996; Seitz & Schumann-Hengsteler, 2002), whilst others have examined executive functions other than inhibition (e.g., Deschuyteneer &

Vandierendonck, 2005a; 2005b) or examined the central executive, referred to inhibition and hinted at its specific type (e.g., Imbo, Vandierendonck & Vergauwe, 2007).

The cognitive processes involved in arithmetic division have received little attention in comparison to the other three arithmetic operations. The few investigations that have been carried out have focussed on simple rather than complex division (e.g., Robinson & Ninowski, 2003; Robinson, Arbuthnott & Gibbons, 2002; Imbo & Vandierendonck, 2007). Division does not play as large a complementary part in solving problems involving the other three operations in that addition and subtraction can be employed to solve multiplication and division, respectively, using multiple blocks of repeated additions and / or subtractions. Moreover, multiplication can be used to solve division problems by exploiting its inverse properties. This does not, in practice, apply to division: it is not used as an inverse function or as blocks of repeated divisions to solve problems requiring addition, subtraction or multiplication; therefore it is less well practised (Robinson & Ninowski, 2003). There is also the problem of the inherent difficulty that adults may encounter when solving division problems; division tends to be the last operation to be taught at school and, in terms of proficiency, it has been found to be the weakest of the four operations (Robinson *et al*, 2002). It is probably because of these characteristics, there has seemingly been some reluctance to study complex division in the past.

Some research has been carried out, in previous literature, with regard to how street children in Brazil perform problems involving division. For example 75 ÷ 5 was thought of as 10 x 5 = 50; that leaves 25 left over; 25 ÷ 5 = 5, so 75 ÷ 5 = 10 + 5 = 15; this was a case of decomposition into multiples of ten and five. In another example, *i.e.* 100 ÷ 4, a child was unable to perform the division using paper and pencil methodology owing to no perception of 1 ÷ 4 or 0 ÷ 4 (quite understandably) but he or she was able to compute the problem mentally by halving and then halving again [½ of 100 = 50; ½ of 50 = 25] (Nunes, Schliemann & Carraher, 1993). The use of multiple fraction sequences is quite a common method for solving such problems in UK schools; however, they are less practicable for more complex division problems such as 1624 ÷ 7.

Within the UK, both metric and imperial measures are employed in everyday life; division can be used in, at least partial fulfilment of, a calculation to convert from one type of measure to another. For example, if we wish to convert 336 km into miles, a very close approximation can be arrived at by multiplying 336 by the fraction five-eighths. This might involve multiplying 336 by 5 (= 1680) and then dividing 1680 by 8 (= 210). Incidentally, it is problems such as 1680 ÷ 8 that will be investigated in the present study. Similar procedures can be found, for example, to convert litres to imperial gallons (multiply by two-ninths) and vice-versa (multiply by nine-halves) or to convert litres to pints (multiply by four-sevenths).

Those who attended school during the latter half of the twentieth century, in England and Wales, would likely have been taught a left to right method for complex division. This may or may not have involved the carrying of remainders, depending on the complexity of the individual problem. For example, a problem such as 1269 ÷ 3 would have been set out as follows:

$$3\overline{)\,1269}$$

The answer would have been placed on the top line and the procedure may well have been: "Three into one won't go, therefore treat the 1 and 2 as 12; three into twelve goes in four times; write 4 above the 2; three into 6 goes twice, write 2 above the 6; finally three into 9 goes three times, write 3 above the 9; the answer is 423". This is an example of a problem with no remainder carrying and is simply a series of short division procedures. Taking a more complex example such as 1352 ÷ 4, the layout would have been the same as the previous example but the procedure would have involved remainder carrying, as follows: "Four into one cannot be done, therefore treat the 1 and 3 as 13; four into 13 goes 3 remainder 1, write 3 above the 3 and put the remaining 1 in front of the 5 to make 15 ($13^{1}52$); four into 15 goes 3 times with 3 left over, write 3 above the 5 and 3 in front of the 2 to make 32 ($13^{1}5^{3}2$); four into 32 goes 8 times, write 8 above the 2; the answer is 338." The small numbers in the brackets represent, what might be termed, remainder carrying. This is the type of methodology the participants for this study were encouraged to employ and the basic theoretical implications of this methodology will be described later. In order to examine prepotent inhibition and its effect on complex division for the first experiment via the dual-task paradigm, a suitable concurrent activity had to be developed.

A recommended method for teaching division to 11 – 14 year olds with mathematical learning difficulties in the USA was suggested by Rivera and Smith (1988). Taking a problem such as 1240 ÷ 5, a similar method to the following was suggested:

$5\overline{)\,1240}$	Will 5 go in to 1 ? No.		
10	Will 5 go in to 12? Yes.	12 ÷ 5 = 2.	12 - (2x5) = 2
24	Will 5 go in to 24? Yes.	24 ÷ 5 = 4	24 – (4x5) = 4
40	Will 5 go in to 40? Yes.	40 ÷ 5 = 8	
	The answer to 1240 ÷ 5 is 248. (Rivera & Smith, 1988).		

There are similarities between the two methods; the latter is probably more conducive to pencil and paper techniques. For the present study, being a study of *mental* arithmetic, the former method was recommended and all participants were encouraged to employ those particular procedures.

From a theoretical point of view, not all such division problems require carrying when a 4-digit dividend is divided by a single-digit divisor. Some require one carry, e.g., 1416 ÷ 3, others require no carries, e.g., 1644 ÷ 4, and others require two, e.g., 1685 ÷ 5. There exists the possibility that the central executive is involved in such a 'number of carries' decision process (Imbo, Vandierendonck & Vergauwe, 2007). Other decisions that need to be made include the value of the carry, using a subtraction procedure; moreover, a decision has to be made regarding partial responses in the form of short division procedures. Procedural intrusions such as not carrying when a carry is required and vice-versa may need to be inhibited (as hinted by Imbo, Vandierendonck & Vergauwe, 2007). On the other hand, intrusions in the form of incorrect responses close to the correct response, or associative confusions may need to be suppressed (Campbell, 1987). If what was hinted at by Imbo, Vandierendonck & Vergauwe (2007) involves the prepotent response inhibition-type, then the present experiment will go at least part-way to clarify this. Furthermore, if PRI is involved in filtering associative confusions, then the present experiment is designed to partially clarify this, but, as will be seen, such clarification is more pertinent to extracting the series of short-division procedures in a separate experimental process (which was carried out for Experiment 3).

A paper which examined inhibitory ability as a specific weakness in teenagers with mathematical disabilities was Barrouillet, Fayol & Lathulière (1997). They studied simple multiplication using a response-selection (multiple-choice) technique. For Experiment 2, participants were instructed to select their answers to a problem such as 8 x 5 from a list of four possible answers under three conditions: non-interference, weak interference and strong interference. Under the non-interference condition, three of the possible answers did not belong to the times-table of either number to be multiplied whereas under the weak interference condition, three of the possible answers were close to the correct answer but not part of either multiplicand's times-table. The three 'distracters' under the strong interference condition were answers from the times-table of either number in the problem. The results, in terms of percentage of correct answers, suggested that most errors occurred within the strong interference condition and that at the beginning of the multiplication process a number of possible responses are cognitively processed, the incorrect ones of which need to be inhibited. The type of inhibition deficit that, according to this paper, affects teenagers with mathematical difficulties might appear to be an internal type related to the confusion between responses that belong to the same times-table as either of the two digits to be multiplied. From a developmental point of view this suggests teenagers with mathematical difficulties display a certain amount of maturational lag with regard to development of this associative-network (memorised number-facts) related inhibition. The amount of influence, or otherwise, of the more externally-based prepotent response inhibition or resistance to distracter interference was not directly examined. Teenagers with mathematical difficulties, according to this paper, may have problems triggering inhibitory control and this may be a causal factor of the problem (Barrouillet *et al*, 1997).

Some more recent studies, however, have cast doubt on the assertion that inhibitory control is actually a cause of arithmetic difficulties, as suggested by Barrouillet *et al* (1997). Censabella & Noël (2004) studied very simple addition and multiplication. The results suggested that both arithmetic operations were more sensitive to association-based interference rather than prepotent response inhibition. In other words, confusion with closely related responses from the cognitive arrays of multiplication and addition facts crystallised in the LTM were the more likely cause of errors. A slightly later paper (Censabella & Noël, 2005) examined prepotent response inhibition and resistance to distracter interference in 11 year old children grouped into controls and those with learning difficulties. The children with learning difficulties did not display significant differences in inhibitory capacity compared with controls. They did, however, yield lower scores on the digits forwards / backwards task, indicating that they had a lower working memory capacity. All three types of inhibition (Friedman & Miyake, 2004) were later studied by the same experimenters (Censabella & Noël, 2008), the participants being 10 year-old children. No significant differences were revealed in terms of inhibitory abilities between the controls and children with mathematical difficulties or between those with specific arithmetic fact retrieval difficulties, further suggesting that there is some unlikelihood in the notion that poor inhibitory capacity is a causal factor with respect to mathematical difficulties.

From more of an applied perspective, inhibitory ability was found to be a factor contributing to algebraic problem solving accuracy in a recent study of 14 year old secondary school pupils in Singapore (Khng & Lee, 2009). Results suggested that those problems that were solved algebraically represented a 76% accuracy rate compared to 42% for those that were solved using arithmetic methods and 70% for those that were solved using a mixture of the two methodologies. Two types of inhibition were highlighted via path analysis following a profile components analysis: inhibition of reified processes and inhibition of recently learned associations. Where inhibition of the more abstract aspects of such mathematical processing was evident (reified processes), this pointed to accuracy through arithmetic intrusions whereas where inhibition of recently learned associations was evident, accuracy through intelligence was suggested (Khng & Lee, 2009). This might suggest that once earlier arithmetic methodologies are thoroughly learned and deeply entrenched within the cognitive processing system, they become more difficult to resist, depending upon the amount of inhibitory control each individual pupil possesses. It also indicates that previously learned methodology may become a source of either prepotent response inhibition or a lack of resistance to distracter information with regard to the acquisition of more advanced abstract methodology. Bull & Scerif (2001) reported similar findings, in terms of inhibition amongst seven-to-nine year old children. Furthermore, the question arises, with regard to the multiple-choice responses manipulated to form distractions in Barrouillet *et al* (1997), as described earlier, are these distractions or prepotent information in terms of 'it is close and it is part of the times-table, therefore it is probably correct?' It is possible to argue either way.

Of interest to the present study and to any possible future educational intervention programmes is the suggestion by Geary & Hoard (2005) that a major underlying reason underpinning mathematical difficulties is poor executive functioning in terms of attentional control and the inhibition of irrelevant associations. Bearing this in mind, could these irrelevant associations be intrusions of information that is easier for the cognitive processing system to extract form the long-term memory in terms of pre-learned number-tables or could they be intrusions of irrelevant or no longer relevant methodologies, either of which might be prepotent or / and a distracter? Before attempting to answer such questions, it seems appropriate to attempt to learn more about the employment by the cognitive processing system of inhibitory mechanisms when applied directly to basic mental arithmetic, using a behavioural paradigm. No future educational support programme could be justified without knowing if any inhibitory mechanisms are directly employed by the cognitive processing system when solving arithmetic division problems.

Research using a behavioural paradigm to try to discover whether or not inhibition, specifically, is involved in mental arithmetic and, if so, to what extent, appears to be somewhat lacking in the literature, to date. There are, however, some beginnings. Fürst and Hitch (2000) studied the effect of central executive suppression on 3-digit addition that involved no carries, one carry and two carries. The Trails Task (in this case, orally alternating the days of the week with the letters of the alphabet) was used as a concurrent activity to suppress the central executive. This central executive suppression had the most dramatic effect on the carrying process; furthermore, problems involving carrying affected the accuracy and fluency of the trails task. A suggested reason was that the strong tendency to carry out additions without carrying, if at all possible – the non-carry state of mind therefore has to be *inhibited* when one is faced with a problem that requires carrying. Imbo, Vandierendonck & De Rammelaere (2007) used the same activity and also a random choice reaction time task (CRT-R) to suppress the central executive; this paper also focussed on complex addition. The CRT-R involved responding vocally ('high' or 'low') to a tone which was either 262Hz or 524Hz sounded, at random, 900 or 1500 ms apart. Similar conclusions were drawn as those for the Fürst & Hitch (2000) study.

There is evidence, therefore that the central executive (CE) is involved in the carrying process with regard to complex addition; multiple additions of the same number becomes multiplication. Seitz & Schumann-Hengsteler (2002) researched simple and complex multiplication under central executive and articulatory suppression. For Experiment 2 the central executive was loaded using a random letter generating task (from the letters a, b, c, d, e and f). In terms of latencies, there was a significant increase in RTs from the neutral condition to the CE suppression condition when participants solved easy multiplication problems, suggesting a major role for the CE in retrieval of number facts from the LTM. Central executive suppression also resulted in significant increases in error rates for complex multiplication problems. Seitz & Schumann-Hengsteler (2002) concluded that the CE was a major force for long-term memory retrieval and monitoring carry-processes. It was also interesting to note references to the random letter generating task as one which interferes with a learned task, presumably reciting letters in alphabetical order; this might be regarded as a type of prepotent response inhibition.

So far, evidence is apparent for central executive involvement in the carrying procedure in both addition and multiplication and for retrieval of facts from LTM in simple multiplication. Imbo, Vandierendonck, & Vergauwe (2007) examined complex subtraction and multiplication under both phonological loads and executive loads. Executive functions were loaded using a similar random choice reaction time task to that employed by Imbo, Vandierendonck and De Rammelaere (2007). It was found that the executive load had a significant effect on latencies and error rates both when compared with the control and phonological loop conditions, and as the number of carry / borrow operations increased. (Carries in multiplication; borrowings in subtraction). Furthermore, more errors were evident on the CRT-R whilst calculating in comparison to when the CRT-R was being performed alone (control). Some evidence, it was suggested, was also present for greater executive involvement the higher the value of the number to be carried. Interestingly, it was suggested that the central executive may be employed to inhibit the normal order of processes, that is calculations without any carry or borrow procedure (Imbo, Vandierendonck & Vergauwe, 2007). When a carry or borrow operation is required, the no-carry cognitive process may need to be inhibited; this was a similar inference to that drawn by Fürst & Hitch (2000).

Some inconclusiveness is evident in the aforementioned studies. With regard to inhibition within the context of arithmetical difficulties, the cause of such difficulties is poor inhibitory control with respect to confusion of number-facts in multiplication or addition networks relevant to pairs of numbers being processed (Barrouillet *et al*, 1997; Geary & Hoard, 2005). Campbell (1987) had already suggested that closely-related but incorrect number facts need to be inhibited in the quest for a correct response. Antithetically, there is evidence that no significant difference in inhibitory control between subjects with and subjects without learning difficulties. Furthermore, arithmetic difficulties may rather be as a result of working memory capacity differences as opposed to poor inhibitory control (Censabella & Noël, 2004, 2005). From a behavioural perspective, it has been suspected that the central executive is involved in the carrying and borrowing procedures in addition, multiplication and subtraction; moreover, the larger the number of carry-operations, the more intensively the central executive works in this monitoring rôle (Imbo, Vandierendonck & Vergauwe, 2007). This leaves unanswered questions. What part of the central executive is involved in monitoring carry operations? Is it prepotent response inhibition and, if so, does it work more intensively as the number of carries increases? These are questions Experiment One is designed to attempt to answer. The question with regard to whether prepotent response inhibition is charged with the filtration of closely related but incorrect responses in each short-division procedure will be probed in experiment three.

3.1 The Stop-Signal Task

Early studies involving what was termed the Stop Signal Task were designed to examine the phenomenon that a primary task can be inhibited if it is closely followed by a second stimulus (Helson & Steger, 1962; Lappin & Eriksen, 1966). The Stop Signal Task consisted of instructing participants to respond to a stimulus light but not to respond if two lights were shown. The two lights varied in terms of stimulus onset asynchrony by 0, 12, 33 or 63ms (Lappin & Eriksen, 1966). It was found that the probability of inhibiting the primary task decreased if the temporal delay between the primary and secondary stimulus was widened. Later research built on this phenomenon and used the second stimulus as a stop signal; participants were instructed to refrain from responding if a stop signal was displayed (or sounded), either simultaneously or very soon after. The results suggested that where subjects actually responded to a stimulus following a stop signal, RTs were lengthened as the stop-signal delays increased (Logan & Cowan, 1984). From a theoretical perspective, it stood to reason that if the cognitive process of responding to the stop-signal finishes first, then the primary task will be inhibited. Antithetically, if the process of responding to the primary task finishes first then the primary task will not be inhibited (Logan, Cowan & Davis, 1984).

The Stop Signal Task was used as a means to assess ‘inhibition of prepotent response information’ by Friedman and Miyake (2004). Their version employed the categorisation of a set of words as either animal or non-animal by means of button-pressing. The words continued to be displayed after a tone was sounded (the stop-signal). The tone was a signal to stop the categorisation activity until participants were instructed to start again. This was known as the stop signal paradigm based on Logan’s work published in 1994 (see Logan, Schachar &Tannock, 1997 for a description of similar methodology). Carter *et al* (2003) developed an updated version: participants were instructed to respond to the direction in which pictures of an aeroplane were pointing (left or right); after a Stop-Man was displayed, they were to stop responding until a green ‘go-signal’ was shown. This paper was aimed at determining whether assessment of the inhibitory abilities of children with ADHD could be improved by setting the stop-signal delays in proportion to individual ‘go’ mean reaction times. It was concluded that both modifications represented an improvement on previous methods such as those used by Logan & Cowan (1984) and Logan *et al* (1997).

EXPERIMENT ONE

The hypotheses, for this experiment were based on the findings of Fürst & Hitch (2000), Imbo, Vandierendonck & De Rammelaere (2007), Imbo**,** Vandierendonck & Vergauwe (2007) and Seitz & Schumann-Hengsteler (2002), it was predicted that prepotent response inhibition, being an executive function, would (1.1) both slow the cognitive process, that is, increase the latencies, and (1.2) increase the error rate, when and only when carrying is required, in division. This would be assuming that the cognitive carrying processes in division may be similar to those of addition and multiplication. If these hypotheses were supported it would suggest that PRI has a responsibility to monitor the carrying procedure but when no carries need to be executed there would be no strong procedural tendencies to inhibit. It was also predicted that the secondary activities, i.e., saying the direction of the arrows in the simultaneous condition, would be (1.3) slowed and (1.4) be subject to increased error-proneness when the problems were being processed, simultaneously; if this were the case, it would indicate the use of other parts of WM such as the articulatory rehearsal system in complex division. Furthermore, (1.5) with regard to the inhibition condition, an increased error-rate in terms of stating the arrow-directions when instructed not to would also be predicted, suggesting an interactive process between inhibiting saying, ‘left/right’ and the complex division procedures.

The focus of the present experiment, and those that follow, was on division problems with 4-digit dividends and single-digit divisors, e.g., 1476 ÷ 9 to result in a three-digit answer. A three-digit dividend would not necessarily have yielded a three-digit response (e.g., 747 ÷ 3 = 249, but 747 ÷ 9 = 83) when larger dividends were used. In this format, all problems consistently have three short division procedures and *up to* two carry procedures depending on the arithmetic condition. This consistency was maintained throughout the present study to investigate the impact of inhibition on calculation procedures and/or the impact of inhibition on retrieval of number facts from LTM procedures. It will also enable the study of the rôle of inhibition on the carrying procedures, where they take place.

The present experiment was designed to first induce a prepotent response in participants by priming them into saying left or right in response to an arrow upon which a division problem was superimposed. The activity used in the first experiment in the present study to load PRI was very loosely based upon Carter's visual version of the stop signal task. Carter *et al* (2003) used a picture of an aeroplane for an experiment designed for child participants; the present experiment was designed for adults, hence it was considered more appropriate to use a picture of an arrow. The expectation was that a habitual response of saying left or right in reaction to an arrow yet simultaneously solving a division problem would be established, this was the simultaneous condition. Once established, the left/right response would be halted by means of a stop-signal – simply a red screen instructing participants to stop saying left or right. However, following the stop-signal, the arrows with problems superimposed upon them would still be present, consequently, participants would have to inhibit the prepotent response of saying left or right; this was the inhibition condition where participants would be expected to engage in inhibiting a prepotent response (Friedman & Miyake, 2004). It ought to be emphasised, however, that although it was expected that the 'inhibitory conditioning' would reduce with time, it was assumed that it would not dissipate completely (see Lappin & Eriksen, 1966). Only under the control (problems) condition would the arrow disappear, and only under the control (direction) condition would the problems be absent but the arrows present again. The 'simultaneous' activity was not examined in the same depth as the 'inhibition' condition; it was an activity that probably tapped input monitoring, response selection, and also phonological resources – it was the 'inhibition' condition that was an attempt to tap prepotent inhibition with some degree of purity.

Although there have been experiments designed to load the central executive, in the past (e.g., De Rammelaere, Stuyven & Vandierendonck, 2001; Fürst & Hitch, 2000; Imbo, Vandierendonck & Vergauwe, 2007; Lemaire, Abdi & Fayol, 1996; Seitz & Schumann-Hengsteler, 2002) the present experiment represents an attempt to extract prepotent response inhibition (PRI), being one of the separable abilities (Friedman & Miyake, 2004) of the stimulus inhibition fraction of the central executive as proposed by Baddeley (1996). This extraction was then formulated into a manipulated quasi-dual task paradigm to impose a load on the cognitive PRI system whilst simultaneously solving division problems. This represents an attempt to discover (a) whether or not PRI is used whilst solving division problems, and (b) the possible role of PRI within the procedural array of subtasks that is tackled when dividing a 4-digit dividend by a single-digit divisor.

Method

From the point of view of the present study, it was felt that it would be much easier for participants to build a strong prepotent response to left or right pointing arrows rather than lists of words belonging to a specific semantic category (as in Friedman & Miyake, 2004). It needed to be borne in mind that participants would have plenty of information to process simultaneously in the form of arrows *and* division problems. It is stressed, also, that the use of visual methodology was of interest in order to maintain consistency of presentation throughout this and the following experiments. Visual manipulations and secondary tasks were used for the following experiments as well as for Experiment One.

Design

The experiment took the form of a 3 (0, 1 and 2 carries) x 3 (simultaneous, inhibition and control) design. The numbers of carries (0, 1 and 2) were the three levels for the arithmetic factor; the simultaneous, inhibition and control conditions applied to the problems were the three levels forming the cognitive factor. The three cognitive conditions, as applied to the vocal direction of the arrows were analysed separately to ascertain whether or not the arithmetic conditions had an effect on participant's vocal responses to the arrows. All conditions were varied entirely within-subjects with response times and error-rates taken as dependent measures. All arithmetic conditions were presented in a pseudo-random manner, i.e., the problems with no carries, one carry and two carries were presented in an order that was randomised in advance. In simple terms, problems without carries, one carry and two carries – with divisors from 2 to 9 were mixed rather than blocked. In this way, all participants solved the problems in the same randomised order to maintain consistency of presentation.

Participants

Forty-three participants volunteered who were recruited from the Schools of Health and Social Sciences, Games Computing and Creative Technologies, Arts, Media and Education, Built Environment and Engineering and Bolton Business School at the University of Bolton. Owing to a large percentage of procedural errors and machine errors throughout the experiment, three sets of data were discarded; furthermore, one decided not to participate following the initial screening process. This left thirty-nine participants, 23 of whom were male and 16 were female; their approximate mean age was 34 years (range, 18 to 65 years). No money was paid to any of the participants.

Stimuli

To maintain consistency of responses and to limit the number / problem size effect, all dividends were greater than 1000 but less than 2000. (The phenomenon that the larger the values in a problem, the longer the RTs are for solving the problem is referred to as the problem size effect [e.g., Campbell & Graham, 1985]). Therefore all began with the digit 1 and all divisors were in the range 2 to 9 inclusive; hence, only a single digit divisor would have to be processed. Moreover, for consistency throughout the experiment, all problems were designed to yield a 3-digit response and were presented in the form of a top-heavy fraction, thus:

$$\frac{1276}{4}$$

For the simultaneous and inhibition conditions, each problem was superimposed, within a white rectangle measuring approximately 75 x 30mm, onto a blue arrow measuring approximately 135mm between the vertical extremities and 280mm between the horizontal extremities. Each arrow was positioned on a white background to the left of the screen, horizontally but within the central area, vertically. The blue arrows pointed either to the left or right, at random. The arrows with superimposed problems were initially composed in PowerPoint, the numbers being in Calibri font, size 18; they were then transferred into Microsoft Paint, into Photo Gallery and then pasted into E-Prime objects. For the control condition involving the problems, division problems were presented using black Courier New text, size 24 at the centre of the screen with no arrows present. For the control condition that just involved responding to the arrows where no problems were present, the same sized arrows were used as in the control and inhibition conditions (but with no problem superimposed on them); furthermore, they were repositioned slightly so they appeared at the top of the screen, horizontally but centrally placed, vertically.

There were 108 division problems in total. To reduce the risk of practice effects, 36 different problems were created for each of three groups: group 1, for the simultaneous condition; group 2, for the inhibition condition; and, group 3, for the control condition (see Appendix I). Within each group, 12 problems were present for each arithmetic condition: no carries, one carry and two carries (see Appendix II). Additionally, 9 further problems were composed for the beginning of the experiment to allow the experimenter to demonstrate the exercises and for participants to practise them.

Apparatus

The stimuli were presented on a Dell Desktop GX 280 computer coupled to a 43cm (17 inch) flat-screen colour monitor. A microphone was used to enable left / right vocal RTs to be collected via an E-Prime serial response box for the simultaneous and control (direction) conditions. E-Prime 2 (Schneider, Eschman & Zuccolotto, 2007) was installed on the equipment and used to control the experiment and collect data.

In E-Prime 2, for the simultaneous condition, each arrow with superimposed problem was presented via a *pair* of ImageDisplay objects. Under the General properties of the first ImageDisplay object of the pair, this was aligned horizontally at ‘left’, aligned vertically at ‘center’ with the Clear After set at ‘No.’ Under Duration / Input, Duration was set at 0, the SRBOX was added along with the Allowable response, 6 and the Correct response, 6; the Time Limit was set at (infinite) and the End Action at (none); only the SR box RT was logged. The second ImageDisplay object was the same as the first with the following exceptions: under the general properties, Clear After was set at ‘Yes,’ under Duration / Input, the Duration was set at (infinite) and the Keyboard was added; the Allowable response option was set at {ANY} and ‘Correct’ was set to the correct response for the problem. The Time Limit was set at (same as duration) and the End Action was set at ‘Terminate.’ These settings enabled RTs for both the manual responses to the division problems and the vocal responses to the arrow-directions to be recorded, simultaneously. Under the Advanced Keyboard Properties, the ‘Max Count’ was set at 3 to enable 3-digit responses to be recorded. Logging was requested for accuracy, correct responses, actual responses and RTs; these RTs were timed up to the entry of the third digit (units) of each manual response. All other properties were set at their default values. No WaitObject was inserted in the middle of these pairs of ImageDisplay objects. In this way, not only could the RTs for the vocal responses to the arrow directions and those for the typed-in answers to the problems be collected simultaneously, but no flicker could be detected as the computer moved from the first to the second ImageDisplay object of each pair. A 500ms blank screen (WaitObject) was inserted after each pair of ImageDisplay objects.

For the inhibition condition, each arrow with superimposed problem was presented via a single ImageDisplay object with the properties set as for the second of each pair under the simultaneous condition.

For the control condition, each problem was presented using a TextDisplay object with the properties set at the same values as for the second of each pair of ImageDisplay objects under the simultaneous condition with the exception of AlignHorizontal under the General properties, which was set at 'center.' A 500ms blank screen was inserted between each TextDisplay object.

For the control (direction) condition where just the direction of the arrows were requested, each arrow was presented using an ImageDisplay object, each separated by a 500ms blank screen. With regard to the General properties, AlignVertical was set at 'top' and Clear After as 'Yes.' Under the Duration / Input properties, Duration was set at (infinite), the SRBOX was added to the devices; under Response Options, Allowable was set at 6, Correct at 6 and the Time Limit as (same as duration). Logging was requested for the RTs only.

Procedure

Participants indicated their age-range and gender on an information pro-forma to consent to take part in the exercise. The experimenter demonstrated the experiment by performing the necessary actions for three examples of problems under each of four conditions: simultaneous, inhibition and control (problems) conditions, followed by three lone arrows under the control (direction) condition. This part of the procedure was repeated to enable participants to practise the exercise in the presence of the experimenter. It also acted as a screening device: the participant and the experimenter briefly discussed whether or not to continue with the experiment-proper based on the strength of the participant's confidence and accuracy when carrying out the division problems. Participants who continued were instructed to press the [SPACE] bar to continue with the main part of the experiment after the experimenter had left the room. Throughout the main part of the experiment, the experimenter stood outside the room and viewed participants' progress through the window in the door. Furthermore, oral responses to the arrow directions could be heard through the door and oral errors to arrow directions and false responses to inhibition condition requests to refrain from responding to arrow directions were recorded on paper by the experimenter; this was to provide error data for any effects on the 'secondary task'

For the first part of the simultaneous condition, participants initially responded to 24 division problems, one at a time, from group 1 (see Appendix I) that had been randomised in advance (see Appendix II) and were superimposed on a blue arrow that pointed either to the left or the right. They were instructed to say, loudly, which direction the arrow was pointing to and then type-in the answer to the problem – hundreds digit first as quickly and as accurately as they could. The next problem did not follow a 500ms blank screen until three digits had been typed in on either the number keys above the letter keys or those on the numeric keypad. At the end of the 24 problems, black text on a red background instructed participants to STOP saying, 'left / right' in response to the arrows and just answer the problems by typing. An instruction at the bottom of the screen asked them to press the [SPACE] bar when they were ready to continue.

Next, the first part of the inhibition condition, a further set of 24 problems from group 2 (see Appendix I) were displayed in a randomised order (see Appendix II), one at a time, still superimposed on a blue arrow. Participants were now expected to type-in the answers to the problems, as before, but to refrain from saying, 'left / right' in response to the arrows and to press the [SPACE] bar when ready to continue. Any responses to the arrows were recorded as errors by the experimenter. At the end of the 24 problems, black writing on a green background instructed participants to resume saying, 'left / right' in response to each arrow and then type-in the answer to each problem, as before. Twelve further problems from group 1 were presented as before, in this, the continuation of the simultaneous condition. At the end of the 12 problems, a further set of instructions on a red background requested participants to STOP saying, 'left / right' and to press the [SPACE] bar to continue. Twelve more problems were displayed from group 2 to complete the inhibition condition.

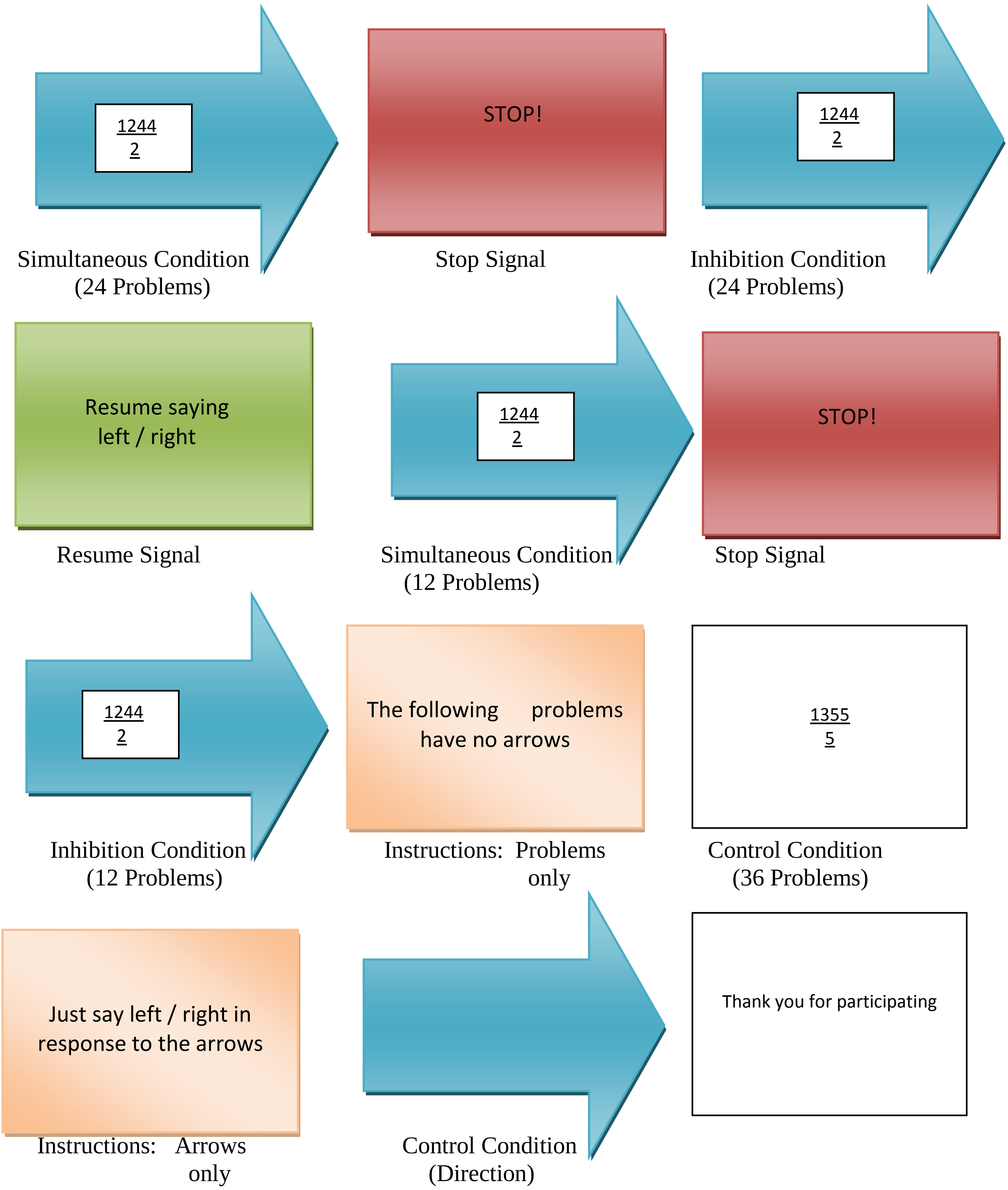

Figure 3.1. Diagrammatic Illustration of the Order of Activities (Experiment 1).

(Note: Arrows pointed either left or right, at random)

For the control condition (problems), a set of instructions on a white background then required participants to type-in answers only to the following problems. Thirty-six problems from group 3 were then presented, one at a time, at random with a 500ms blank-screen in between each problem. The computer did not display the next problem until a 3-digit answer was typed-in.

The final section, the control (direction) condition, consisted of a yellow instruction screen with black text asking participants to respond, ‘left / right’ to the following 36 arrows. These were presented one at a time on a white background with a 500ms blank screen in between each arrow; the computer moved to the next arrow as soon as a vocal response was detected. A final ‘Thank you’ screen was then displayed. Any errors were recorded by the experimenter. See figure 1 for a pictorial representation.

Results

Only the RTs of the correctly answered problems were analysed. Under the simultaneous condition, the latencies for any problems that were coupled with errors in stating the direction of the arrows were removed from the data. With respect to the inhibition condition, those RTs coupled with a response situation to the arrows were also removed from the analysis.

Owing to machine errors because of the microphone not recording vocal responses, phonological rebound or coughs, 8.47% of the voice-related data were discarded. Of the remaining data, only the latencies of the correctly stated directions paired with correctly answered division problems were analysed. Under the control (direction) condition, because participants performed this part of the exercise with considerable speed, all RTs were analysed after the removal of machine errors.

The remaining data were screened participant by participant and any values that were ±2 standard deviations away from the mean were replaced by the mean. This process was undertaken for the problem RTs in the simultaneous, inhibition and control (problems) conditions and for the direction latencies under the simultaneous and control (direction) conditions. Overall, 1.8% of the RT data with respect to answers to the problems and 2.3% of the vocal RT (direction response) data were replaced in this way.

Response Times

Analysis in the form of a 3 x 3 ANOVA revealed a significant main effect of the cognitive factor $F(2,76) = 6.63$, $p = 0.002$, $\eta^2_p = 0.15$, reflecting a general slowing under simultaneous and inhibition loads, particularly when carrying was required A significant main effect was also indicated for the arithmetic factor (0, 1 and 2 carries), $F(2, 76) = 175.98$, $p < 0.001$, $\eta^2_p = 0.82$, indicating the intensified difficulty of the problems as the number of carries increased.

Moreover, a significant interaction between the cognitive and arithmetic factors was revealed, $F(4, 152) = 7.96$, $p < 0.001$, $\eta^2_p = 0.17$, suggesting either the inhibition or simultaneous or both conditions had more effect as the number of carries increased.

Examination of Table 3.1 provides a clearer indication as to the relationship between pairs of mean values and where the interactions occur. As the ANOVA only reveals the overall effects of the experimental conditions, a series of *post hoc* tests were carried out on control *versus* inhibition, control *versus* simultaneous and inhibition *versus* simultaneous values to further clarify these relationships. Nine pairs of means were therefore compared; as such multiple comparisons are likely to lead to Type I Errors, a Bonferroni-type stepwise correction was applied beginning with the lowest *p* value, as recommended by Benjamini & Hochberg (1995). In this way, possible side effects of Type II Errors occurring and reduction of power are alleviated (Benjamini & Hochberg, 1995; Verhoeven, Simonsen & McIntyre, 2005). Analytically, significant increases or significant decreases in RTs were specifically sought after, in these *post hoc* tests, rather than overall upward or downward effects; hence all tests were one-tailed.

Table 3.1

Mean RTs (in milliseconds) for Problems (N = 39)

Cognitive Condition	No. of Carries	M	SD
	0	5056	1465
Control	1	8578	3244
	2	13 950	5897
	0	4943	1405
Inhibition	1	9644	3616
	2	14 839	5190
	0	5796	1550
Simultaneous	1	8707	3164
	2	12 991	4399

Regarding the inhibition condition, as a reminder, it was predicted that there would be a significant slowing of latencies when carrying had to be enacted. Where no carries were required, there was no significant increase in RTs ($p = 0.21$). The one carry condition resulted in a significant increase, $t(38) = -2.71$, $p = 0.005$, $\alpha = 0.02$; the two-carries condition also resulted in an increase in RTs that was significant, $t(38) = -1.93$, $p = 0.0305$, $\alpha = 0.039$. These results supported the predictions that RTs would only increase when carrying was required (hypothesis 1.1) and also suggest that the interaction was caused by the significant increase in latencies when carrying took place, coupled with no significant change in latencies when no carries were required.

As the simultaneous condition involved a dual task resulting in divided attention and was not necessarily the main focus of this study, this will be examined last. Under the no-carry arithmetic condition, there was a significant increase in RTs from the control to the simultaneous condition, $t(38) = -4.87$, $p < 0.001$, $\alpha = 0.006$. When one carry was required the increase was not significant $p = 0.308$, however, there was a significant decrease when the problems demanded two carries, $t(38) = 1.99$, $p = 0.027$, $\alpha = 0.03$. Comparing the effects, from the inhibition to the simultaneous conditions, where no carrying was required, there was no significant effect, $p > 0.05$; with one carry, there was a significant decrease, $t(38) = 2.00$, $p = 0.027$, $\alpha = 0.028$. With respect to the two-carry condition, there was also a significant decrease in RTs from the inhibition to the simultaneous conditions $t(38) = 4.56$, $p = 0.03$, $\alpha = 0.02$. There was a far greater effect on latencies caused by the inhibition condition than by the simultaneous condition, indicating a more intensive load on the cognitive system when participants had to inhibit saying, 'left/right' in response to the arrows; this, in turn, suggested a successful loading of the prepotent response inhibition system by using a manipulative (simultaneous) activity immediately prior to the main focus of the experiment.

Error Rates

A 3 x 3 ANOVA was performed on the error rates for the problems under the three arithmetic and three cognitive conditions. This revealed no significant main effect of the cognitive factor, $F < 1$, $p > 0.05$, indicating that either the secondary activities did not provide a sufficient cognitive load in order to cause an increase in errors or that prepotent response inhibition did not have a monitoring role in terms of filtering numerical errors. The ANOVA did, however, reveal a significant main effect of the arithmetic factor (0, 1 and 2 carries), $F(2,76) = 56.67$, $p < 0.001$, $\eta^2_p = 0.66$, suggesting an increase in error-rate in-line with greater problem difficulty. Also a significant cognitive factor x arithmetic factor interaction was evident, $F(4, 152) = 4.61$, $p = 0.002$, $\eta^2_p = 0.11$, reflecting the main effect of the number of carries but no main effect of the cognitive factors. Table 3.2 provides clarification as to between which pairs of conditions the interactions occur.

Table 3.2

Mean Errors (%) (Problems)

Cognitive Condition	Arithmetic Condition	M	SD
Control	0 Carries	9.40	10.68
	1 Carry	19.02	12.96
	2 Carries	26.07	17.43
Inhibition	0 Carries	6.41	9.65
	1 Carry	18.38	13.68
	2 Carries	35.47	21.52
Simultaneous	0 Carries	8.76	8.96
	1 Carry	18.80	16.53
	2 Carries	31.41	22.66

At first sight, the hypothesis that PRI would lead to an increase in error-rates has been refuted. To confirm this or otherwise, a series of *post hoc* tests were carried out to separate any significant changes in error rates between cognitive conditions when zero, one and two carrying operations were required. The only significant comparison was that of control *versus* inhibition when two carry operations were required, $t(38) = -3.78$, $p = 0.0005$, $\alpha = 0.006$ (one-tailed). This represents a significant *increase* in error rate, suggesting a possible numerical rôle for prepotent response inhibition with respect to the two-carry procedure; this partially supported the hypothesis that PRI would intensify error-proneness. Furthermore, this was the root of the interaction and this reflected a possibility that PRI was involved in monitoring carry procedures when the problems were at their most difficult. None of the other comparisons reached significance ($p > \alpha$, in all cases).

This could be interpreted as PRI fulfilling a responsibility in terms of considering and filtering procedures that involve no carries, or just one, when two carries were required.

Individual Differences

In order to refute or otherwise the possibility of individual differences being a likely extraneous variable, the control error-data were examined and separated into two groups. Group 1 consisted of 23 participants who made three errors or fewer in each arithmetic condition and group 2 were the remaining sixteen. The RT and error data from the Control and PRI cognitive conditions were subjected to a MANCOVA, group 1 entered as the covariate and like-for-like data-sets compared (e.g., control group 1, no carries with the corresponding condition in group 2, and so on). No significant differences were apparent, hence, participant strength with respect to executing division problems was unlikely to be an extraneous variable.

Secondary Task (Saying the Direction of the Arrows)

Hypothesis 1.3 predicted that the latencies of saying the direction of the arrows would be slowed when it was used as a secondary task performed simultaneously with solving the problems. The vocal response times for the stating of the arrow-directions were analysed by means of an ANOVA which indicated a significant main effect of arithmetic factor in comparison with control [saying the direction of the arrows with no superimposed problems], $F(3, 114) = 156.97$, $p < 0.001$, $\eta^2_p = 0.81$. Examination of Table 3.3 suggests that the main difference is between the control condition and each of the arithmetic conditions (0, 1 and 2 carries). A series of *post-hoc* tests (one-tailed) with partial Bonferroni stepwise corrections indicated significant increases in RTs for control vs. no-carries, control vs. one-carry and control vs. two carries, t (38) = -15.46, $p < 0.001$, $t(38) = -14.20$, $p < 0.001$ and t (38) = -14.51, $p < 0.001$, respectively (α values: 0.008, 0.017 and 0.025, respectively). When pairing no carries with two carries and one carry with two carries, significant differences between RTs were revealed, $t(38) = 3.07$, $p = 0.02$, $\alpha = 0.042$; and $t(38) = 1.94$, $p = 0.03$, $\alpha = 0.05$, respectively. In the case of no carries *versus* one carry, this only approached significance, $p = 0.038$, $\alpha = 0.0332$.

Table 3.3

Mean Latencies (Direction-Voice) [ms] under the Arithmetic Conditions (N = 39)

Levels	M	SD
Control	511	83.14
No Carries	1061	254.28
One Carry	1022	259.35
Two Carries	985	243.04

With regard to the direction latencies, it was evident that the arithmetic condition of the problem did have an impact on the cognitive processing of the direction of each arrow: there was a slight increase in the speed of processing as the number of carries increased – the opposite way to what one might expect (see Table 3.4). The difference in RTs between the control (direction) condition and the simultaneous condition as a whole suggests that the arithmetic process began as soon as the problem (superimposed on an arrow) appeared. Furthermore, these results suggest less likelihood of speed-accuracy trade-off between arithmetic calculating and stating the arrow directions.

Table 3.4

Mean RTs for Directions (ms)

Cognitive Condition	Arithmetic Condition	M	SD
Control	N/A	490	61.90
	0	967	206.96
Simultaneous	1	927	201.68
	2	912	221.96

To further investigate this indication, a Pearson's Correlation analysis was carried out on the mean latencies of the problems versus those of the responses to the arrow-directions to further check for trade-offs in performance. None of the correlations were significant: $r = 0.17$, $p = 0.31$; $r = 0.24$, $p = 0.14$; and $r = 0.04$, $p = 0.81$ for no carries, one carry and two carries, respectively (two-tailed), confirming no speed-accuracy trade-off. Owing to the nature of the inhibition condition, similar analyses were not practicable with regard to the inhibition condition as it was not, strictly speaking a dual-task condition. Furthermore, the general direction of the latencies was the same, that is, either relatively stable or increasing, when the control and the prepotent-inhibition conditions are compared. The hypothesis that solving the problems would slow the vocal responses to the arrow direction was therefore supported.

Direction Errors

The error-rate for stating the direction of the arrows or nil-responses under the simultaneous (direction), inhibition and control conditions was low and spread amongst few participants. As it was assumed that these results were not parametric, two-tailed Mann-Whitney U tests were carried out on pairs of conditions. No significant difference was found between the Control and Inhibition conditions, $U = 724.50$, $p = 0.57$, between the Control and Simultaneous conditions, $U = 658.50$, $p = 0.18$ or between the inhibition and simultaneous conditions, $U = 627.50$, $p = 0.066$. This did not support the predictions that the secondary task would become more error-prone.

Discussion

Complex division, as illustrated in the introduction, consists of a number of procedures. The main purpose of the present experiment was to discover whether or not prepotent response inhibition (PRI), as an executive ability, forms part of the cognitive processing of complex division problems. It was predicted, on the basis of the findings of Fürst & Hitch (2000), Imbo, Vandierendonck & De Rammelaere (2007), Imbo**,** Vandierendonck & Vergauwe (2007) and Seitz & Schumann-Hengsteler (2002), that the inhibition condition would slow the latencies and elevate the error rates, particularly as the number of carry operations increased. Overall, their findings have been supported by the results of this experiment but some differences in the detail were noticeable.

Under these particular experimental conditions, PRI made no significant difference in the speed of cognitive processing when no carrying had to be implemented but did slow the process down when carrying was required. This is consistent with the prediction and suggests that the suppression activity successfully separated prepotent inhibition from memory-updating which is believed to be more involved in arithmetic that does not involve carrying. Of particular note is that, when carrying was required, inhibition slowed the process more than the simultaneous section of the experiment which, on the surface at least, might be expected to have caused a greater division of attention owing to the phonological, visual and response-selection aspects of this dual-task activity. Also, of particular note, is the decrease in RTs from the control to the simultaneous condition when two carries are required; this could suggest that a dual-task condition acts as an aid to processing with two carrying manoeuvres and reduces cognitive load.

More specifically, hypothesis 1.1 stated the expectation that PRI would slow the cognitive processes when carrying was undertaken. There was an overall main effect of the cognitive factor in the ANOVA. When the post-hoc tests were performed, there was indeed a significant increase in latencies under the PRI condition in comparison with the control condition when one carry was implemented and when two carries took place. In contrast, when no carries were required, there was no significant difference between the RTs when comparing the control with the PRI conditions. The initial notion would be to suggest that, as PRI only caused interference when carrying was required, then this particular type of inhibition has a rôle in monitoring the carrying process as hinted at by Fürst & Hitch (2000) and Imbo, Vandierendonck & Vergauwe, (2007). When no carrying is required there is a strong possibility that participants simply produced a series of three simple division procedures in order to attain the final response and the results indicate no interference in this case, suggesting that such short division procedures may come directly from LTM, bypassing the central executive. Another possible interpretation is that, if the central executive is not bypassed, this no-carry procedure may be more reliant on the memory-updating fraction rather than response inhibition. Memory updating is considered to be a link to LTM (*cf*. Baddeley, 1996).

Hypothesis 1.2 referred to an increase in error-rates as a result of PRI. The overall main effect in the ANOVA was not significant although there was a significant interaction between the cognitive and arithmetic factors. The *post hoc* tests revealed only one significant comparison, i.e., the increase in error rates under the PRI condition compared with the control condition when *two* carries were undertaken. No other comparison was significant; moreover, this was the cause of the interaction. The problems with two carries were the most difficult ones; this is where there was an effect of PRI. The second hypothesis was therefore partially supported. The reason for the lack of effect on the 'one-carry' condition may have been because this condition lacked sufficient difficulty to be monitored by PRI; on the other hand, if the 'one-carry' condition requires monitoring, an executive ability other than PRI is allocated to this condition. Friedman & Miyake (2004) suggest quite emphatically that executive abilities are separable but not independent; this leaves open the possibility that more than one executive ability may be involved in monitoring carrying procedures. Not only did Friedman & Miyake (2004) suggest that executive abilities were separable but not independent but also that prepotent response inhibition and resistance to distracter interference (RDI) correlated, i.e., they worked together. Taking this separabilty and correlation together, this exposes the possibility that RDI and PRI may work together to monitor carrying procedures. For obvious reasons such an assertion cannot be supported or refuted, here, but the information collected in the next chapter will illuminate matters in this respect.

Hypotheses 1.3 and 1.4 referred to the secondary task of saying the direction of the arrows whilst simultaneously solving the division problems. This was of less interest in the present study because this simultaneous condition was designed to induce a prepotent response to be inhibited during the PRI condition. It will nevertheless be discussed, briefly. Contrary to the findings of Imbo, Vandierendonck & De Rammelaere (2007) and Imbo**,** Vandierendonck & Vergauwe (2007), it can be argued, that the secondary task in the present study (the simultaneous condition), was significantly disturbed by solving the division problems, in terms of latencies (hypothesis 1.3). However, this was not the case, in terms of error-rates. Because the simultaneous activity involved vocal responses to the direction of the arrows it is a strong possibility that it loaded the phonological loop. Moreover, it involved deciding the direction of the each arrow, suggesting it probably loaded response selection. This is at least two working memory components or subcomponents. One also needs to bear in mind that as well as loading the central executive with the CRT-R, Imbo, Vandierendonck & De Rammelaere (2007) suppressed inhibition by employing the 'Trails Task,' in-line with Fürst and Hitch (2000). Participants were shown a day of the week and a letter of the alphabet, both at random and were asked to recite the sequence (i.e., Wednesday, G → Thursday, H; Friday, I; .....); when the end of the sequence (presumably the alphabetical sequence), they were asked to start at the beginning again: (Sunday, A; Monday, B; ....). This was an activity that tapped into phonological resources as well as prepotent response inhibition and was probably considerably more difficult, as a secondary task than the one used in the present study. The Trails Task would also involve memory updating, particularly when subjects came to the end of the alphabet sequence; it also may have involved switching (or response-selection), that is, continuously alternating between 'days of the week' mode and alphabetic mode. Consequently, the Trails Task not only causes disturbance by breaking the prepotent tendency to match the first seven letters of the alphabet, sequentially, to each day of the week but also loads memory updating and the phonological loop, at least. The main point is that the Trails Task and the simultaneous task in the present study both loaded more than one working memory component or subcomponent, the Trails Task being the more cognitively demanding of the two.

This posed the problem of purity: a secondary task that loads more subcomponents of WM than is needed. The present study has attempted to address this by using the dual task (the simultaneous condition) as a manipulative device to induce the required condition (the inhibition condition), the latter being the primary focus of the study. It can only be inevitable that an elaborate secondary task such as the Trails Task will create more disturbances of primary and secondary tasks, in comparison with the inhibition condition in the present study.

The fifth and final hypothesis that error-rates would increase for the secondary task when simultaneously solving the problems was not supported neither was there any significant difference in direction-stating error rates during the PRI condition. This may have been owing to the secondary task being of relatively low cognitive demand in comparison to the semantic categorisation task used by Friedman & Miyake (2004).

In conclusion, from the evidence provided by Experiment One, PRI can be regarded as a type of inhibition that has a supervisory rôle in the monitoring of the carrying process and plays a part in filtering errors if, and only if, the problem is demanding enough. It would therefore have a stronger procedural responsibility and perhaps a weaker arithmetic or numerical responsibility. What is meant, here, by procedural responsibility is monitoring carry procedures whereas arithmetic responsibility refers to checking that numerical responses are the intended ones. Campbell & Clarke (1989) when studying simple multiplication suggested that responses to problems such as 3 x 7 are drawn from a cognitive network of number-facts (tables-chart), the closer ones to the correct response of which needed to be inhibited. One might therefore expect there to have been a slowing of RTs and an increase in error rates when *no carries* were required if PRI had a specific arithmetic rôle such as filtering incorrect responses that are close to the intended response; moreover if there were any possibility of this occurring it would need to be investigated by designing a study with the intention of extracting the series of short division procedures from each problem and investigating the effect of PRI on these.

A contrasting possibility that needs to be discussed is the fact that when solving problems with carries, and particularly when using the methodology participants were encouraged to employ, in the case of two carries (the only condition where error proneness was apparent, in the present experiment), the first two out of the three partial responses are approximate. To clarify: a no-carry problem such as 1869 ÷ 3 was solved by a series of short-division sub-problems, 18 ÷ 3, 6÷ 3 and 9 ÷ 3; these all demand exact answers. A two-carry problem such as 1947 ÷ 3 was solved by amalgamating 19 ÷ 3, 14 ÷ 3 and 27 ÷ 3, the first two of which demand approximate answers, i.e., 6 and 4 respectively. It is not apparent from the literature on simple arithmetic if there exists a cognitive network of number-facts for such approximate responses; it is very unlikely that there is. One does need to bear in mind, however, that a problem such as 17 ÷ 3 has more than one response. If an integer response is required, then the response is 5, if a response within the context of the number of three-seat vehicles is required to transport 17 persons is expected, then the answer is 6; a figure that is rounded to the nearest whole number. And if an exact response is required then the response is $5\frac{2}{3}$. It may be the case that two of the responses interfere with the intended response and these need to be inhibited. Such a theory would be consistent with Khng & Lee (2009); where an individual has a procedural tendency to respond with a type of answer within a dominant context then this may need to be inhibited; for example, a person who has a strong tendency to provide exact responses would need to inhibit $5\frac{2}{3}$ in order to successfully respond with 5 to the problem, 17 ÷ 3. To filter such dominance is a logical responsibility for prepotent response inhibition.

Finally, one cause for concern might be the question regarding the arrow itself: there is a strong possibility, as can be observed from the elevated RTs during the inhibition condition, that the simultaneous condition induced a prepotent response that participants had to inhibit. There is a chance, however, that the arrow surrounding each problem in the inhibition condition may have been a distraction. It has been emphasised that Friedman & Miyake (2004) maintained that prepotent response inhibition and resistance to distracter interference (RDI) were correlated and perhaps worked together. The results from the next experiment, which was designed to focus on RDI, were intended to help clarify this issue.

CHAPTER FOUR

The Effect of Resistance to Distracter Interference on Mental Arithmetic Division

The second experiment was designed to follow and complement the results and conclusions reached in the first experiment. Friedman & Miyake (2004) provided strong correlational evidence in their factor analytical study that resistance to distracter interference is closely related to prepotent response inhibition. Both types of inhibition have shared requirements in that resisting interference and resisting dominant but unwanted intrusions demand that the primary task being executed (in the case of the present study, a division problem) needs to be kept in a high state of activation (Friedman & Miyake, 2004). It has been proposed in Chapter 3 that PRI has a procedural responsibility in terms of monitoring carrying procedures and this was consistent with Imbo and colleagues (Imbo, Vandierendonck & Vergauwe, 2007) hint that the strong tendency to solve a problem without carrying may need to be inhibited where appropriate. Moreover, in the previous chapter, PRI load increased error-proneness but only when problems demanded two carrying procedures; the possibility was raised that this is one instance where PRI works together with another type of inhibition.

Resistance to Distracter Interference (RDI) is defined as the ability to suppress unwanted external intrusions from working memory (Friedman & Miyake, 2004). From an arithmetic point of view Passolunghi, Cornoldi & De Liberto (1999) studied children with poor problem solving abilities. When solving a problem such as: "A pizza costs 8500 lire and a drink costs 2500 lire; what is the bill and how much change should be given in exchange for a 50 000 lire banknote?" The results suggested that proficient problem solvers tended to focus on the prices whereas poor problem solvers paid more attention to the irrelevant information, the irrelevant information being the pizza and drink. The theoretical implication here is that children with stronger executive functioning in terms of stronger interference control solved problems such as these more efficiently.

4.1 The Eriksen Flanker Task

The Eriksen Flanker Task (Eriksen & Eriksen, 1974) was originally created to investigate the effects of noise letters on the identification of target letters on a cathode-ray tube screen and is thought to employ the resistance to distracter information system (Friedman & Miyake, 2004). Eriksen and Eriksen (1974) produced six experimental conditions: the control condition, where the target letter was shown alone; the compatible condition, containing letters either all angular or all curved letters, e.g.: KKK**H**KKK; the incompatible condition containing an angular target letter with curved noise letters or vice-versa, e.g.: SSS**H**SSS; the heterogeneous-similar condition where the target letter was flanked by letters with similar attributes (angular or curved), e.g.: NWZ**H**ZWN; the heterogeneous-dissimilar condition where the target letter was adjacent to letters with dissimilar attributes, e.g., GJQ**H**QJG; and a condition where the noise letters were the same as the target letter, e.g., HHH**H**HHH. There was a significant increase in RTs from control (and noise-the-same-as-target) to 'noise' conditions and also a significant increase in error rates. It was proposed that these effects were as a result of response competition or interference between the target letters and flankers; furthermore, both the target letters, and also the flankers took some processing energy, hence the elevated RTs. They also varied the spacing between the letters and found that the interference effect reduced as the letters were placed further apart. When the spacing was commensurate with normal writing (0.06 degree of visual angle, as opposed to 0.5 or 1 degree for those wider apart), it was suggested that participants processed target and flanker letters simultaneously and some form of inhibitory process was activated to prevent processing of the flankers (Eriksen & Eriksen, 1974).

The Eriksen Flanker task has latterly been adapted as a task for studying resistance to distracter interference in digit naming. The present experiment represents a further extension of the Task for the purpose of studying complex division. For this second experiment, a numerical form of flanker task was created for the 'Resistance to Distracter Information' (RDI) condition, similar to those employed by Notebaert & Verguts (2006), Censabella & Noël (2005) and Ullsperger, Bylsma & Botvinick (2005) as digit naming tasks and by Nuerk and colleagues (Nuerk, Bauer, Krummenacher, Heller & Willmes, 2005) as a magnitude verification task. They used a numerical flanker task consisting of seven digits under compatible (e.g., 222**2**222) and incompatible (e.g., 222**1**222) conditions (Censabella & Noël, 2005; Ullsperger *et al*, 2005) whilst Nuerk *et al* (2005) classified theirs as identical (e.g., 2222222), congruent (e.g., 2221222; 2 and 1 are in the range 1 to 4) or incongruent (e.g., 2227222; 7 is in the range 6 to 9). Notebaert & Verguts (2006) used two flanker digits on either side of the target and studied the effect of numerical distance between target and flankers (e.g., 11911: distance of 8; 11311: distance of 2). The present study represents an extension of this technique into the study of complex division.

The 'top-heavy' fraction layout of the division problems in the present study lent itself to this type of format and consequently the relevant digits forming the problems themselves (the targets) were naturally underlined. This is the first time, as far as is known at the time of writing, that flanker digits have been employed to cause interference when solving complex division problems. Eriksen and Eriksen (1974) based their letter classification on the Gibson System (Gibson, 1969) which classified capital letters according to whether they were angular or curved and a number of other features such as horizontal, vertical or slanting lines (see Gibson, 1969: p. 88 for more detail). It was more difficult to classify the ten digits in this way, for the present study; hence, what was considered by the experimenter to be the best contrast or incompatibility in terms of 'noise digits' were used and is further explained in the method section. Eriksen & Eriksen (1974) found that when the letters were half a degree of visual angle or more apart, it was relatively easy for participants to ignore the flankers. For this reason, no variation of the spacing between the digits was implemented in the present study; they were typed adjacent to each other as if they were inside a word, the intention being to provide maximum distraction.

Bearing in mind that this experiment was also designed to activate a type of inhibition process, resistance to distracter interference, which is an executive function, and on the basis of the findings of Fürst & Hitch (2000), Imbo, Vandierendonck & De Rammelaere (2007), Imbo, Vandierendonck & Vergauwe (2007) and Seitz & Schumann-Hengsteler (2002) with the addition of the evidence from Eriksen & Eriksen (1974), it was predicted that the presence of flanker (noise) digits would increase both RTs and error rates when solving the problems both when carrying takes place and when it does not. If this hypothesis were supported it would suggest that the flanker digits successfully created response competition between the problem and the flanker digits. It was also predicted, on the basis of the findings of Friedman & Miyake (2004) that both PRI and RDI would work together to filter interferences such as prepotent tendencies and possible distractions such as close but inappropriate responses.

EXPERIMENT TWO

Experiment Two was designed to test the following hypotheses: 2.1) The latencies under the RDI condition would be longer in comparison with the control condition, including when no carrying takes place; 2.2) the error rate would significantly increase under the RDI condition, regardless of the number of carries; 2.3) the latency and error data when compared between Experiments One and Two would suggest that PRI and RDI worked together; and 2.4) that there would be significant differences between the RT and error data from Experiments One and Two.

Method

For Experiment Two it was decided that the most practicable activity with a view to tapping into resistance to distracter interference was to use a numerical version of the Eriksen flanker task (Eriksen & Eriksen, 1974) with three noise digits rather than noise letters on either side of the dividend and the divisor. Friedman & Miyake (2004) originally used their own version that was closer to the original task: with three noise letters on either side of the target letter.

Design

The experiment took the form of a 3 (0, 1 and 2 carries) x 2 (control and RDI) design. The numbers of carries (0, 1 and 2) formed the arithmetic factor; the control and RDI conditions formed the cognitive factor. All conditions were varied entirely within-subjects with response times and error-rates taken as dependent measures. All arithmetic problems were presented in a pseudo-random manner, i.e., the problems with no carries, one carry and two carries were presented in an order that was randomised in advance, as for Experiment One. The problems for the control condition were the same as those for the corresponding condition in Experiment One; those for the RDI condition were the same as those used for the PRI condition in Experiment One.

Participants

Thirty-four participants volunteered to take part in the experiment, none of whom had participated in Experiment 1; twenty-one were male and thirteen were female. Their approximate mean age was 25 years with a range of 18 to 55. All were undergraduate students from the Schools within the University of Bolton, as described in Experiment 1. No participants decided against participation following the initial screening process. Participants were not paid any money.

Apparatus and Stimuli

The same computer system was used as for Experiment 1 but without the serial response box and microphone; these two items were not necessary for this Experiment. The division problems for the control condition were the same as those used in corresponding condition in the first experiment. The problems for the RDI condition were the same as those used during Experiment 1 for the inhibition condition. Problems were presented in the same order and in the same top-heavy fraction format. Seventy-two problems were therefore presented, thirty-six from group 2 (RDI) and the same number from group 3 (control) [See Appendix I]. Additionally, six problems, three in each cognitive condition, were presented at the beginning of the experiment for demonstrative and practice purposes. For the RDI condition, the problems were presented with three contrasting digits on either side of the dividend and on either side of the divisor (See Figure 4.1).

Control Format	RDI Format
1276	2221388222
4	2224222

Figure 4.1. Examples of the Control and RDI Formats for the Division Problems

The contrasting digits in the RDI condition were different from all those within each problem. They were originally typed in Microsoft PowerPoint (2007) in Calibri (Body) text, font size 24 and then copied into Microsoft Paint which automatically transferred each problem as a jpeg file in PhotoGallery. The PhotoGallery images were then programmed as ImageDisplayObjects into E-Prime 2. Within E-Prime 2, under General Properties, Align Horizontal was set at ‘right’ and Align Vertical at ‘bottom.’ This was to adjust the positioning of the problems so they would be displayed in the centre of the computer screen.

For the control condition, the stimuli were presented as TextDisplay Objects in Courier New text, font size 24, underlined and emboldened; under trial, this provided the clearest image. The E-Prime properties in the ‘General’ section were set as follows: AlignHorizontal at ‘center,’ AlignVertical at ‘center,’ to help centralise the images. Under the Duration / Input section, the Duration property was set at ‘infinite’ to allow the image to remain on the screen until the participants had entered a complete three-digit response. The ‘keyboard’ was added to enable responses to be collected. Under ‘Response Options: Keyboard,’ ‘Allowable’ was set at {ANY}; the ‘correct’ answer was inserted; ‘Time Limit’ was set at (same as duration), again, this was to allow complete three-digit responses to be typed before the computer moved on to the next problem. In the Keyboard Advanced Properties, ‘Max Count’ was set at 3 to enable three-digit answers to be collected. Other properties were left at their default settings. For the RDI condition, under General Properties, ‘AlignHorizontal’ was set at ‘right,’ AlignVertical at ‘bottom;’ this was done to centralise the slightly different images. Other settings were the same as for the control condition.

Procedure

Participants indicated their age-range and gender on an information pro-forma and signed a consent form to take part in the study. The experimenter demonstrated the procedure by performing the necessary actions for three examples of problems under each of the two conditions: RDI and control. This part of the procedure was repeated to enable participants to practise the exercise in the presence of the experimenter and ask any appropriate questions. It also acted as a screening device: the participant and the experimenter briefly discussed whether or not to continue with the experiment proper based on the strength of the participant's accuracy and confidence when carrying out the division problems. Participants were instructed by the computer to press the [SPACE] bar to continue with the main part of the experiment after the experimenter had left the room.

The main part of the experiment consisted of seventy-two problems, thirty-six in each condition. The first set of 18 problems was under the RDI condition after participants were instructed by the computer to ignore the noise digits and focus on each problem itself and type in the answers as quickly and as accurately as they could, starting with the 'hundreds' digit. The computer did not move to the next problem after a 500ms blank screen until three digits (hundreds, decades and units) were typed. Participants were then informed that the next set of problems would not have noise digits and they were to just type in the answers as quickly and as accurately as they could. Twelve problems followed under the RDI condition, twelve under the control condition, six under RDI and the final six followed under the control condition, making six sets of problems in the main part of the experiment, each preceded by the relevant instructions. The order of the problems was randomised in advance both in terms of arithmetic condition and value of the divisors; hence all participants solved the problems in the same randomised order (see Appendix II).

Results

Response Times

Only the RTs of the correctly solved problems were analysed. The remaining data were screened subject by subject and latencies that were ±2 standard deviations away from the mean were replaced by the mean. Four per-cent of the data were replaced in this way. The means and SDs are displayed in Table 4.1.

Table 4.1.

Mean Latencies (ms) and Standard Deviations (N = 34)

Condition	M	SD
Control (No Carries)	5629	3035.09
Control (One Carry)	9645	4352.39
Control (Two Carries)	15 584	8058.24
RDI (No Carries)	6405	3835.05
RDI (One Carry)	11 448	7607.74
RDI (Two Carries)	18 401	9036.21

A 2 (control vs. RDI – the cognitive factor) x 3 (0 vs.1 vs 2 carries - the arithmetic factor) repeated measures ANOVA revealed a significant main effect of RDI on RTs, $F(1,33) = 21.45$, $p < 0.001$, $\eta^2_p = 0.39$, suggesting significant interference when flanker digits were present. There was also a significant main effect of arithmetic factor (no carries, one carry, two carries), $F(2,66) = 111.77$, $p < 0.001$, $\eta^2_p = 0.77$ reflecting the slowing of processing as the difficulty of the problems intensified. There was no significant cognitive-factor x arithmetic-factor interaction, $p > 0.1$. The main effect of arithmetic factor was expected and can be attributed to increased complexity owing to the number of carries.

Three *post hoc* paired samples t-tests were carried out to clarify significant differences or otherwise on the mean RTs of control versus RDI under the no, one and two-carry conditions. As only significant *increases* in RTs were being tested for, all tests were one-tailed. Consistent with Experiment 1, a stepwise partial Bonferroni correction was implemented to reduce the risk of Type I and Type II errors. These results are reported in Table 4.2 and, as can be seen, revealed significant increases in latencies, regardless of the number of carrying operations, when problems were flanked by noise digits. This was the prediction stated in hypothesis 2.1.

Table 4.2.

Post Hoc Tests (RTs)

Comparison	α	Result (One-Tailed)
Control v. RDI (no carries)	0.0167	$t(33) = -4.04$, $p < 0.001$
Control v. RDI (two carries)	0.033	$t(33) = -3.57$, $p = 0.001$
Control v. RDI (one carry)	0.05	$t(33) = -2.18$, $p = 0.018$

Note: RDI, Resistance to Distracter Interference.

Errors

A 2 (control and RDI) x 3 (0,1 and 2 carries) ANOVA revealed a significant main effect of cognitive factor (control and RDI) on the percentage error rates, $F(1,33) = 7.02$, $p = 0.012$, $\eta^2_p = 0.18$. There was also a significant main effect of arithmetic factor (0, 1 and 2 carries), $F(2,66) = 21.56$, $p < 0.001$, $\eta^2_p = 0.40$. As with RTs, there was no significant interaction between the arithmetic and cognitive factors, $F(2, 66) = 0.99$, $p = 0.38$, indicating that differences in error-rates between control and RDI were similar, regardless of the number of carries. Table 4.3 displays the general pattern of errors under the cognitive and arithmetic factors.

Table 4.3

Mean Error Rates (%) and Standard Deviations (N = 34)

Condition	Mean	SD
Control (No Carries)	9.56	12.16
Control (One Carry)	17.16	12.82
Control (Two Carries)	24.02	22.45
RDI (No Carries)	6.13	10.72
RDI (One Carry)	9.80	11.87
RDI (Two Carries)	21.32	15.92

Post hoc tests were carried out to clarify whether the decreases in error rates were significant or otherwise on the mean percentage-error rates of control versus RDI under each of the zero, one and two-carry conditions. Again, a stepwise partial Bonferroni correction was implemented to reduce the risk of Type I and Type II errors; these results are reported in Table 4.4. There was one significant result: control versus RDI when one carry was required, although one might argue that the comparison under the 'no-carry' condition approached significance. The results did not support hypothesis 2 that there would be a significant increase in error rates. The overall trend was a *reduction* in error-rates as a result of RDI. This, coupled with the RT analysis, suggested *speed-accuracy trade-offs* of varying degrees in each of the arithmetic conditions. Additional analyses were therefore implemented.

Table 4.4

Post Hoc Tests (Percentage Errors)

Comparison	α	Result (One-Tailed)
Control v. RDI (one carry)	0.0167	$t(33) = 3.23$, $p = 0.0015$ (sig.)
Control v. RDI (no carries)	0.033	$t(33) = 1.87$, $p = 0.035$ (app. sig.)
Control v. RDI (two carries)	0.05	$t(33) = 0.76$, $p = 0.228$ (ns.)

Note: app. sig.: approached significance

Additional Analyses

Speed-accuracy trade-off (SAT) analyses, as specific experimental and analytical techniques, have, in the past, involved choice reaction time tasks such as stating the colour of a red or green circle or stating the position of a line (left or right). They have involved a large number of trials and relatively short dependent RTs of around 500ms (Arieh & Marks, 2008; Lappin & Disch, 1972a, 1972b, 1973; Wickelgren, 1977). If the error data, in the present experiment, were presented as a function of RTs, they would not form a straight line as in the case of Lappin & Disch (1972a, 1972b, 1973) and latencies were considerably longer than 500ms. Neither would they form a hypothetical SAT function for asymptotic data, as suggested by Pachella (1974) or Wickelgren (1977). By asymptotic, what is meant is the accuracy rate plateaus at a certain response time and does not improve further, even under slower response times. Bijleveld, Custers & Aarts (2010) compared participants' performance on rewarded arithmetic verification tasks, in terms of speed-accuracy trade-off but used ANOVAs as the form of analysis to compare SATs at different reward levels. For their Experiment 2 they decomposed the interactions in the main 3 x 2 ANOVA by implementing two 2 x 2 ANOVAs to compare rewards above the level of consciousness, and below. A similar type of analysis was carried out with the present data for this second experiment, modelled on that of Bijleveld *et al* (2010) to compare SAT characteristics under different numbers of carries, with some variation on how the data were screened. Also, by decomposing the main effects it was possible to search for any parallel attributes between cognitive and arithmetic factors and any opposing characteristics between the behaviour of the latencies and the accuracy rates.

In line with Bijleveld *et al* (2010), only the RTs of the correctly answered problems were analysed, however their data were screened in a less radical fashion: any data that was 3 SDs above the mean was removed from their analysis. To maintain continuity in the present study, the previous screening based on ±2 SDs was retained. For this speed-accuracy trade-off analysis, error-rates were subtracted from 100 to form accuracy rates (following Bijleveld *et al*, 2010). Figure 4.2 shows that latencies look elevated in similar proportions, regardless of the number of carries, when the flanker digits were present (RDI condition). Furthermore, latencies under both control and RDI conditions slow as the difficulty of the problems intensify (i.e., the number of carries increases), in almost a parallel fashion. Figure 4.3 illustrates the improved accuracy rates when the flanker digits were present; the extent of the improvement was similar, regardless of the number of carries. The overall fall in accuracy rates as the problem difficulty increased was both notable and expected.

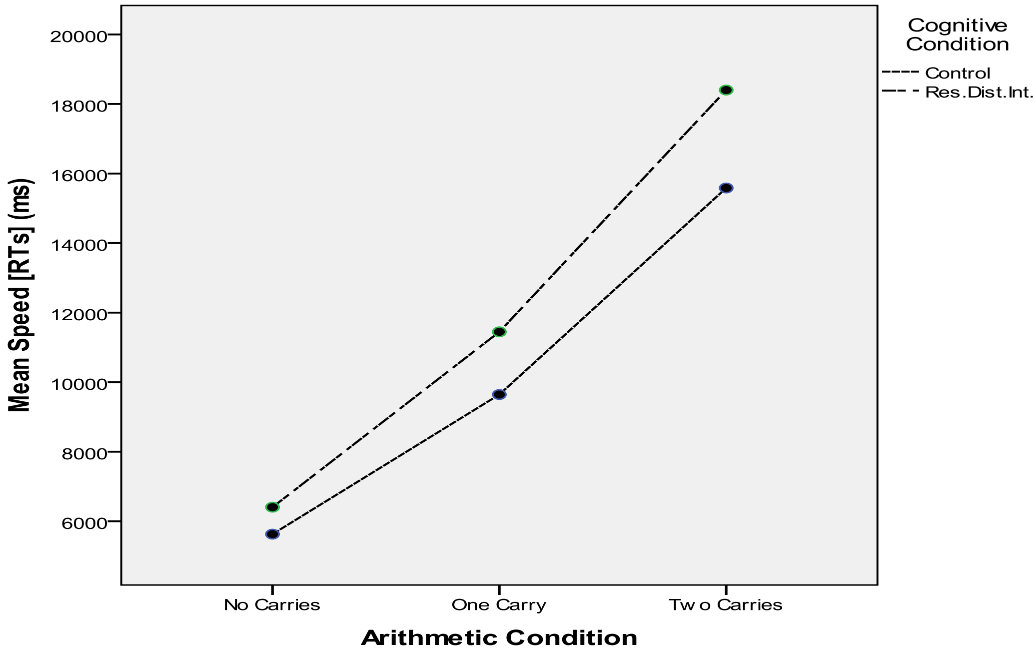

Figure 4.2. Speed (Mean RTs) in ms

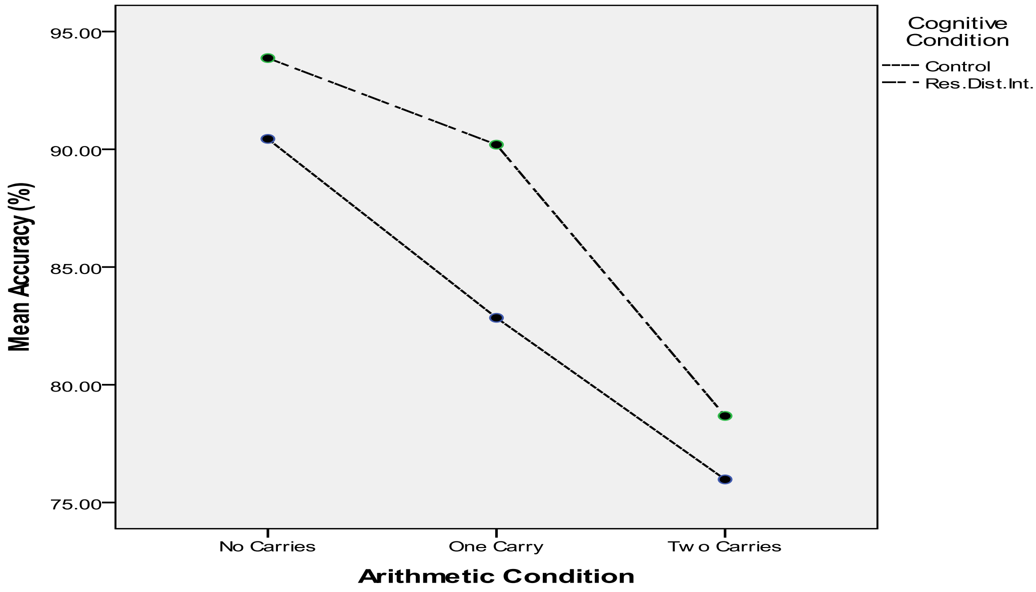

Figure 4.3. Mean Accuracy (%)

Speed

A 2 (control, RDI) x 3 (0, 1 & 2 carries) repeated measures ANOVA revealed the same significant main effect of both cognitive and arithmetic factors on RTs, with the same *F*, *p* and η^2_p values as reported under *Response Times*. Neither was there any significant cognitive-factor x arithmetic-factor interaction, $p > 0.1$ The main effects were decomposed by carrying out two 2 (cognitive factor) x 2 (arithmetic factor) ANOVAs in order to look for any separate main effects and possible interactions. The first ANOVA, a 2 (control vs. RDI) x 2 (no carries vs. one carry) analysis revealed a significant main effect of arithmetic factor, $F(1,33) = 7.43$, $p = 0.01$, $\eta^2_p = 0.18$ and there was a significant main effect of RDI, $F(1, 33) = 79.09$, $p < 0.001$, $\eta^2_p = 0.71$. The interaction approached significance ($p = 0.056$), indicating a slightly stronger effect of RDI on one carry. The second ANOVA, a 2 (control, RDI) x 2 (no carries, two carries) analysis revealed a significant main effect of RDI, $F (1,33) = 19.59$ $p < 0.001$, $\eta^2_p = 0.37$ and a significant main effect of the number of carries, $F(1, 33) = 118.37$, $p < 0.001$, $\eta^2_p = 0.78$. The interaction was significant, $F(1,33) = 6.32$, $p = 0.017$, $\eta^2_p = 0.16$, indicating a stronger effect of RDI on two carries. The greater the number of carries, the more intensive was the slowing effect of RDI. Another point of interest, however, is the considerably larger partial eta squared value, as a result of RDI on one carry, indicating quite a large effect of RDI in terms of proportion of variance.

To take the analysis a step further than was implemented by Bijleveld *et al* (2010), series of *post hoc* tests (one-tailed) comparing differences between no carries and one/two carries. revealed four significant comparisons (see Table 4.5) confirming the proposition that latencies were elevated more intensively as the number of carries rose and when the flanker digits were present (RDI condition). Furthermore, latencies under both control and RDI conditions slowed as the difficulty of the problems intensified (i.e., the number of carries increased). No correction was implemented as it would have made no difference owing to the low p values (see Table 4.5).

Accuracy

Figure 4.3 displays the overall means for the accuracy rate. In line with Bijleveld *et al* (2010), *accuracy* rates rather than error rates were analysed in the form of a 3 (0 vs. 1 vs. 2 carries) x 2 (control vs. RDI) repeated measures ANOVA. The overall analysis revealed a significant main effect of the number of carries with no significant interaction; the statistical values are reported in the *Errors* section.

As with speed, both the main effects were decomposed using two 2 x 2 ANOVAs. A 2 (no carries vs. one carry) x 2 (control vs. RDI) ANOVA yielded a significant main effect of the number of carries, F (1, 33) = 9.59, $p = 0.004$, $\eta^2_p = 0.23$. A significant main effect of RDI was also revealed, $F(1, 33) = 8.42$, $p = 0.007$, $\eta^2_p = 0.20$; no significant interaction was reported, $p >$ 0.05. A 2 (no carries vs. two carries) x 2 (control vs. RDI) ANOVA indicated there was a significant main effect of arithmetic factor, $F(1, 33) = 27.00$, $p < 0.01$, . $\eta^2_p = 0.45$. However, there was no significant main effect of RDI, $F(1, 33) = 2.31$, $p = 0.14$, neither was there a significant cognitive factor x arithmetic factor interaction, $p = 0.86$, suggesting a close to parallel non-significant effect of RDI when comparing no carries with two carries. The accuracy data were subjected to a series of *post hoc* tests with partial stepwise Bonferroni correction (see Table 4.6) and indicated four significant comparisons. In conjunction with an examination of Figure 4.3, this suggests that RDI did result in a less intensive reduction in accuracy when comparing no carries with one carry. This may be explained by reiterating that it was on one carry where there was a significant reduction as a result of RDI but one has to bear in mind that the reduction in error rate on no carries was marginally significant.

Table 4.5.

Post hoc tests (Speed)

Comparison	α	Result (One-Tailed)
Control (0 carry vs. 1 carry)	N/A	$t(33) = -8.70$, $p < 0.001$
Control (0 carry vs. 2 carry)	N/A	$t(33) = -9.43$, $p < 0.001$
RDI (0 carry vs. 1 carry)	N/A	$t(33) = -6.64$, $p < 0.001$
RDI (0 carry vs. 2 carry)	N/A	$t(33) = -10.72$, $p < 0.001$

Table 4.6.

Post hoc tests (Accuracy)

Comparison	α	Result (One-Tailed)
Control (0 carry vs. 1 carry)	0.0375	$t(33) = 3.11$, $p = 0.002$ (sig.)
Control (0 carry vs. 2 carry)	0.025	$t(33) = 3.53$, $p = 0.0005$ (sig.)
RDI (0 carry vs. 1 carry)	0.05	$t(33) = 1.90$, $p = 0.033$ (sig.)
RDI (0 carry vs. 2 carry)	0.0125	$t(33) = 5.55$, $p < 0.001$ (sig.)

Overall, there was a speed-accuracy trade-off as a result of RDI that was particularly significant for the one-carry condition, indicating this was where participants were most cautious when the flanker digits were present. The SAT was less intensive for no carries and even less so for two carries.

Comparison between Experiments 1 and 2

Recent latent variable analysis evidence has suggested that central executive functions are separable but not independent (Fournier-Vincent, Larigauderie & Gaonac'h, 2008) and prepotent response inhibition and resistance to distracter interference are correlated (Friedman & Miyake, 2004). For this reason, an attempt was made to investigate how closely together, or otherwise, the two types of inhibition may operate particularly with regard to their different effects, as dictated by the number of carry-operations required. From the results, so far, from both Experiments 1 and 2, PRI and RDI, each have a different rôle with regard to the processing of complex division problems, hence if the data from both experiments are subjected to a between-subjects analysis, there should be a significant difference between them, particularly when carrying is required. The following comparisons were carried out to test the hypotheses that (2.3, as previously stated) PRI and RDI would work together and (2.4) that there would be significant differences between the RT and error data from Experiments One and Two. Should these hypotheses be supported, it would suggest that PRI and RDI work closely together but they nevertheless have differing rôles.

A 2 (PRI vs. RDI) x 3 (0,1and 2 carries) ANOVA with the two types of inhibition treated as between-groups variables was performed, firstly on RTs. This revealed a significant main effect of the number of carries $F(1,71) = 218.66$, $p < 0.001$, $\eta^2_p = 0.75$, but no significant between-subjects main effect ($p > 0.05$). There was no significant arithmetic-factor x cognitive-factor interaction, $p > 0.10$, suggesting that RDI and PRI did not work together (hence refuting hypothesis 2.3). However, RDI slowed RTs consistently more than PRI, regardless of the problem difficulty, hence, supporting hypothesis 2.4.

The same type of analysis carried out on the error-rates indicated a significant main effect of arithmetic-factor, $F(2,142) = 67.41$, $p < 0.001$, $\eta^2_p = 0.49$, and of cognitive factor, $F(1,71) = 8.83$, $p = 0.004$, $\eta^2_p = 0.11$. A significant cognitive-condition x arithmetic-condition interaction was also recorded, $F(2,142) = 6.51$, $p = 0.003$, $\eta^2_p = 0.08$. This represents partial support for the hypothesis (2.3) that PRI and RDI work together, in terms of error-rates.

The arithmetic conditions were examined separately in order to ascertain the differences in effects on different numbers of carries between the two types of inhibition. One way ANOVAs were implemented on each arithmetic condition, firstly on latencies and secondly on error-rates. To ascertain individual effect sizes of the differences between PRI and RDI for each arithmetic condition, Cohen's *d* (Cohen, 1988) was calculated using the pooled standard deviations from PRI and RDI; the formula used was based on that recommended by Rosnow & Rosenthal (1996) with pooled standard deviations:

$$d = \frac{M_{PRI} - M_{RDI}}{\sqrt{((\sigma^2_{PRI} + \sigma^2_{RDI})/2)}}.$$

Beginning with RTs, the analysis on the no-carry condition revealed a significant difference between PRI and RDI, in the form of an elevation in latencies with regard to no-carries, $F(1,71) = 4.92$, $p = 0.03$, $d = -0.36$; for one carry, no significant difference was evident, $F(1,71) = 1.75$,

p = 0.19, d = -0.214. However, the difference (elevation) was significant for two-carries, $F(1,71) = 4.40$, $p = 0.04$, = -0.342. Regarding error-rates, there was no significant difference, in terms of reduction in errors under the no-carry condition, $F(1,71) = 0.14$, $p = 0.91$, $d = 0.02$. The differences (reductions) were significant, however, for one-carry, $F(1,71) = 8.06$, $p = 0.006$, $d = 0.474$, and two-carries, $F(1,71) = 9.94$, $p = 0.002$, $d = 0.53$. Note the low effect sizes regarding latencies and the moderate effect sizes with respect to differences in error-rates. Regarding RTs there is a significant but small difference in the effect of PRI and RDI with regard to no carries and two carries. There is, however, a moderate sized difference with respect to error-rates regarding one and two carries. Differing rôles are suggested for PRI and RDI from these results and the suggestion is more pronounced in the error data. This partially supports hypothesis 2.4 that PRI and RDI work together; this collaboration depends on the difficulty level of the problem.

Table 4.7

Means and Standard Deviations for RTs and Error-Rates (PRI vs. RDI) [for comparison]

RTs (ms)			Errors (%)		
PRI			*PRI*		
	Mean	*SD*		*Mean*	*SD*
No Carries	4943	1405	No Carries	6.41	9.65
One Carry	9644	3616	One Carry	18.38	13.68
Two Carries	14 839	5190	Two Carries	35.47	21.52
RDI			*RDI*		
	Mean	*SD*		*Mean*	*SD*
No Carries	6405	3835	No Carries	6.13	10.72
One Carry	11 448	7608	One Carry	9.80	11.87
Two Carries	18 401	9036	Two Carries	21.32	15.92

To establish whether or not variations in results were owing to participants' individual differences in their ability to undertake division problems, participants were separated into two variables. One group of data formed the stronger variable and numbered eighteen participants who made three errors or fewer in each of the arithmetic conditions (no, one or two carries) under the control condition; the rest formed the weaker variable. A MANCOVA was carried out with the stronger variable as the covariate. No significant between-subjects difference was evident when comparing each like-for-like condition. Furthermore, participant-cohorts in experiments 1 and 2 solved exactly the same problems, in the same order, and under the same experimental conditions when the ‘control’ problems were solved. Two 2 x 3 ANOVAs were conducted on the pooled control-condition data from both Experiments 1 and 2. ([Experiment 1 vs. Experiment 2] x [0,1 and 2 carries]: Experiments 1 and 2 treated as between-subjects variables). No significant differences were revealed between the two cohorts in terms of latencies or error-rates, regardless of the number of carry operations and no significant interaction was evident, $p > 0.05$. It is therefore unlikely that any of the phenomena reported in the results could be a manifestation of participants’ individual differences in arithmetic ability.

Discussion

At this stage, the results from Experiment Two suggest an important rôle for resistance to distracter interference within the working memory processing system when executing such complex division problems in that although processing time is lengthened considerably, in certain cases, accuracy is improved. In all cases, the trend is towards a general improvement in accuracy.

The results of the initial RT analyses were consistent with the findings of Fürst & Hitch (2000), Imbo, Vandierendonck & De Rammelaere (2007), Imbo**,** Vandierendonck & Vergauwe (2007) and Seitz & Schumann-Hengsteler (2002), in that latencies increased when the flanker digits were present and when carry operations took place (hypothesis 2.1). It does, however, need to be borne in mind that these papers did not attempt to separate different inhibitory abilities as has been attempted in this study but proposed that inhibition may be the subcomponent of the central executive that was responsible for monitoring carrying procedures. In the case of RDI, however, there were also elevated latencies when no carries were needed, suggesting that RDI may have a rôle in monitoring partial responses taken directly from LTM. Just to reiterate, when no carries need to take place, the division problems in the present study are simply a series of three short-division problems. There is evidence that responses to small division problems (e.g., 12 ÷ 2; 8 ÷ 2; 6 ÷ 2) are retrieved directly from a network of division facts in LTM (Campbell, 1999). Later work, however, suggests that division problems might be processed by either direct retrieval or inverse reference to multiplication facts depending on the skill level or preference of participants (Campbell & Alberts, 2010). Either way, direct retrieval from a network of division facts, i.e., 12 ÷ 2 = *6* or mediation of division by inverse reference to multiplication facts, i.e., 2 x *6* = 12, might involve accessing a cognitive network of number facts. When faced with a pair of numbers, e.g., 3 and 6 these trigger a number of associated number facts such as 18 or 9, depending on whether 3 and 6 are multiplied or added (LeFevre, Bisanz, & Mrkonjic, 1988). It may also be the case that faced with a problem such as 3 x 6, associated number facts e.g., 15 and 21, as well as 18 are activated (Campbell & Clarke, 1989). For the correct response to be selected the incorrect but associated responses need to be inhibited; this might be a responsibility or part-responsibility for RDI.

Figure 4.2, coupled with the results of the post hoc t-tests on the RTs, show that the differences between control and RDI progressed in the same direction. That is upwards, from control to RDI, with slopes steepening somewhat as the carry operations increase. This was further clarified by the two separate 2 x 2 ANOVAs reported under the *Speed* section when comparing speed-accuracy trade-offs for different numbers of carries. For no-carries versus one-carry there was an interaction that approached significance owing to the more intensive effect of RDI on one carry. For no-carries versus two carries, the interaction was significant, indicating an even more intensive effect, in terms of slowed latencies on two carries. From a theoretical perspective, this indicates that RDI is a type of inhibition that is *sensitive* to problem difficulty and suggests an inhibitory mechanism that is proactive and not only is it sensitive to problem difficulty but, further, it also continuously monitors problem difficulty.

The results from the initial ANOVA and *post-hoc* tests on the error data refuted the second hypothesis that errors would increase when the flanker digits were present. The results in the first error analysis, i.e., the reductions in error-rates from control to RDI, were somewhat unexpected. There was, however, only a significant reduction for the one-carry arithmetic condition (*cf. post hoc* tests, Table 4.4), although it might be argued that when no carries were required the effect approached significance ($p = 0.035$; $\alpha = 0.033$). The speed-accuracy trade-off for two carries, taking the *post-hoc* results into account, is not really reliable. The two 2 x 2 ANOVAs in the *Accuracy* section provided confirmation that the most reliable speed-accuracy trade-off occurred when one-carry procedure was required. Overall the results from the error data suggested that RDI is an inhibitory mechanism that is *selective* in that it becomes more active according to the amount of interference encountered. This raises the question as to what causes the selectivity and why RDI was more active in reducing errors when one carry was needed rather than two. The results also indicate that the priority for RDI was to reduce interference in problems with one carry, followed by no-carries, and little intervention, if any was necessary for problems requiring two carries.

There may be a number of reasons for these unexpected results. One reason may be that the participants who volunteered for these experiments were confident at solving division problems. It could be assumed that none had difficulties with inhibitory control – however, none were tested for this. It is feasible that the flanker digits triggered resistance to distracter interference resulting in participants becoming more focussed on each problem, hence making fewer errors. The considerably longer latencies caused by RDI when compared to those caused by PRI load, in the previous experiment, strongly suggest that RDI might have resulted in the immediate onset of interference control. Not only was there the immediate onset but it probably remained present throughout each division procedure. When the original flanker task was discussed (Eriksen & Eriksen, 1974), it was proposed that the flanker letters and target letter were processed simultaneously and some form of inhibitory process was activated to prevent the processing of the flankers. One might argue that, in the present study, the flanker *digits* led to participants processing the flanker digits and the problem simultaneously, hence activating RDI. The problem, at first sight, with this argument is that because the task was to solve a multi-procedural problem, one might expect this effect to be present at the beginning of the division process – the encoding stage. The evidence against the flankers just affecting the encoding stage, however, is provided by the extended RTs and the notable SAT, particularly regarding one carry and, to a lesser extent, no carries. The original Eriksen flanker task was aimed at studying letter recognition and categorisation. The present study represents an extension of this into causing interference with a multiple procedure culminating in solving an arithmetic problem. The evidence presented in the present experiment suggests once the conflict between flanker digits and problem has been inhibited at the encoding stage, the inhibitory effect at least, is maintained throughout the problem. This proposition adds weight to the earlier suggestions that flanker digits trigger RDI and, further, that RDI is a proactive mechanism that is sensitive to problem difficulty, i.e., that can be intensified as and when required.

Speed-Accuracy trade-off has formerly been associated with decision making. If participants are instructed to focus on accuracy, often with an instructional limit on the number of permissible errors, then latencies will lengthen. If participants are instructed to focus on speed and respond as quickly as possible, then although RTs will reduce, error-rates will increase (Smith, 1968). An example of previous research has involved changing the instructional bias in this way or has provided differing rewards for a correct response, e.g., 50c vs. 1c (Bijleveld *et al* 2010). Such a speed-accuracy trade-off in the context of the present study and with respect to implementing arithmetic division procedures when problems are flanked by noise-digits cannot, at this stage be attributed to instructional variations: none were present. Participants were plainly instructed to respond to each problem as quickly and as accurately as they could.

In the search for further insights, one might return to the Eriksen Flanker Task (Eriksen & Eriksen, 1974) itself. This has been revisited, empirically, on numerous occasions since it was first established, leading to several theoretical notions, including the Response Competition Paradigm (Eriksen, 1995), the Conflict Monitoring Theory (Botvinick, Braver, Barch, Carter &

Cohen, 2001; Notebaert & Verguts, 2006) and the Gratton Effect (Davelaar & Stevens, 2009). Other studies have examined the effects of orientation of flanker symbols (Hommel, 2003), applied flanker effects to spatial perception with regard to estimating the position of horizontal line bisectors (Fischer & Stumpp, 2001) or have examined visual directional perception (Sanders & Lamens, 2002).

Eriksen (1995) suggested that a rise in RTs in the presence of stimuli that are incompatible (e.g., those with an angular target letter coupled with curved flankers [UUUAUUU]) may be owing to reciprocal inhibition – a term first introduced by Sherrington in a series of lectures in 1904 (cited by Burke, 2007). Such terminology originally referred to the simultaneous opposing forces of groups of muscles at skeletal joints and may be less applicable to the cognitive processing of division problems. In studying the Response Competition Paradigm, Notebaert & Verguts (2006) experimented with target-flanker distances in a numerical flanker task and found that greater target-flanker distances (e.g., 11911) produced lower RTs whereas shorter distances (e.g., 88988) resulted in increased RTs. In fact, RTs decreased as target-flanker distances increased. It was suggested that increased latencies were owing to a stronger focus on relevant dimensions. Because the problems in the present study contained a variety of digits, it was not practicable to vary the numerical distances; flanker digits were chosen because they were not present in the problem. The proposition that there was a stronger focus on the relevant dimensions, in the case of the present study, the problems is plausible and would, at least partially explain the extended RTs and reduced error-rates.

Conflict monitoring theory suggests that response conflict triggers conflict adaption in that the conflict monitoring system assesses the intensity of the conflict and transfers this information to the cognitive centres responsible for control; consequently, these centres adjust the strength of their influence on the cognitive processing system (Botvinick *et al* 2001). Periods of elevated conflict may lead to a reduction in response priming resulting in slower but more accurate responses – a speed-accuracy trade-off. Response priming can occur not only after errors but also after correct responses that involve a high degree of conflict (Botvinick *et al*, 2001). If any response priming in the present experiment occurred, it was owing to the order of the problems. These were presented in a randomised order, hence a problem requiring two carries would have been preceded by a problem requiring no carries or one carry, at random; one might term this procedure priming in that, for example, if one solves a problem involving two carries followed by one requiring only one carry, then the two-carries procedure would need to be inhibited. Just to digress slightly it has been cited that faced with a problem such as 3 x 6, associated number facts e.g., 15 and 21, as well as 18 are activated (Campbell & Clarke, 1989). For the correct response to be selected, certain associated responses need to be inhibited and this might be at least a part-responsibility for RDI. Selection of relevant number facts may be a rôle allocated to the response selection sub-component of the central executive (Deschuyteneer *et al*, 2006); there is a suggestion here that inhibition and response selection work together to process a correct response. It is plausible that response selection may be involved in selecting particular methods, e.g., to carry or not to carry and is supported, if required by an inhibitory mechanism, possibly RDI – particularly if an inappropriate method forms a distraction or conflict. Conflict monitoring theory is a possible candidate for explaining the effects of RDI.

A further point of interest on examining later flanker-task literature is with regard to incipient activation of responses. Conflict caused by flankers may result in partial (irrelevant) information being transferred to the response system leading to response competition, particularly in the initial stages of formulating a response (Eriksen, 1995). This is feasible in terms of fast responses to choice reaction-time tasks but may only be a small part, if at all, of the cause of SAT within the context of multidigit responses to division problems.

Hypothesis 2.3 maintained that both the RT and error data would suggest that PRI and RDI worked together. Results, however, only partially supported this. This was not wholly consistent with the factor-analytical evidence of Friedman & Miyake (2004) but was set within a completely different context: a multi-procedural arithmetic process. The activities used by Friedman & Miyake (2004) involved activities such as shape-matching, word naming and letter categorisation that demanded relatively short response times. Examination of separate arithmetic procedures such as short division will inherently have shorter reaction times owing to reduced cognitive demand and may reveal completely different correlational results. By separate arithmetic procedures what is meant is treating, for example, each short division procedure (e.g., $12 \div 2$, $6 \div 2$ and $4 \div 2$ for $1264 \div 2$) as separate sub-problems and examining the effect of RDI and PRI on them. The results in the comparison between Experiments 1 and 2 do suggest that the two types of inhibition do not operate in the same manner even if they should work together. Extracting separate arithmetic procedures would shed more light on this.

From comparing RDI with PRI, it follows that RDI slowed latencies more than PRI but the between-subjects ANOVA (RTs) revealed no interaction suggesting that RDI and PRI do not work together from the perspective of latencies. From the perspective of error-rates, these increased under PRI and decreased under RDI. However, on examination of the comparison between RTs, the only differences yielded by the one-way ANOVAs were in the no-carry and two-carry conditions; moreover, the size of all the differences was small (cf. *d*-values), casting some questionability on the reliability of these results. There was a moderate size of the differences with respect to error-rates when carrying takes place, as could be inferred from the *d* values suggesting a greater rôle for RDI in monitoring errors. Errors, however, could be thought of as numerical, i.e., incorrect responses, or procedural, e.g., attempting to solve a problem demanding carrying with a non-carry procedure. At this stage it remains an unanswered question as to the type of errors being monitored.

In summary, the flanker digits may have resulted in caution, possibly triggered by some form of cerebral conflict monitoring that, in turn, was triggered by intensive neural control mechanisms, which led to slower but more accurate responses. Bearing in mind that this experiment was not a CRT study in that three-digit responses were required and, moreover, there existed one correct response and many incorrect ones, the question arises as to which of the multiple procedures (all, some, or none) might be most affected by any conflict monitoring mechanism within the whole complex division process. Moreover, this raises the question as to which procedure it might be. Another question remains unanswered, either partially or completely. This concerns the nature and timing of the two types of inhibition. From the evidence provided in these experiments, it looks plausible that, for accurate arithmetic division to take place resistance to distracter interference should be implemented first. The design of the experiment, however, ensured that it did; and prepotent response inhibition needs to work reactively throughout the calculation procedure. The results from the present experiment suggest that RDI was triggered and worked proactively throughout the division procedure. This can be inferred from the extended latencies, in comparison with those generated by PRI. It seems more likely, however, that PRI may work throughout the division process but is more reactive in that it is activated as and when needed. In contrast, although RDI is very likely proactive, it is sensitive to problem difficulty and has the ability to intensify, as the need arises. Ideally all the separate procedures within the process of complex division should be separated and studied as dissociated arithmetic activities. This was the intention of the following experiments.

Two obvious candidates for these separate activities for study were the short division procedures and those of remainder-carrying: for example, 1424 ÷ 4 may be proceduralised as 14 ÷ 4, 22 ÷ 4 and 24 ÷ 4 for the short divisions; whereas 14 ÷ 4 leaves a remainder of 2; 22 ÷ 4 leaves 2; and 24 ÷ 4 leaves 0, for the carrying procedure. The next chapter will focus on the first of these procedures, namely, the short-division procedure in order to establish the effects and timing of PRI and RDI on this particular procedure and how these effects relate to the whole complex division process.

CHAPTER FIVE

The Effect of Prepotent Response Inhibition and Resistance to Distracter Interference on the Short Division Procedures within the Complex Division Process

Stimulus inhibition is one element of the central executive, an attentional control system that co-ordinates three slave systems contained within working memory (Baddeley, 2000). It had been proposed, earlier, that the central executive could be fractionated into four sub-components: an input monitor, for co-ordinating two simultaneous tasks; memory updating for extracting relevant information from LTM; response selection, for retrieving pertinent methodologies for a particular task or sub-task; and stimulus inhibition for suppressing unwanted information (Baddeley, 1996). Recent neurological studies have suggested that the term inhibition has been used too broadly in that it was regarded primarily as a mechanism for inhibiting unwanted informational intrusions and was really a general term covering, more specifically: pre-potent response inhibition, inhibition of previously required information that is no longer required – resistance to proactive interference, and resistance to distracter interference, for filtering unwanted external intrusions from working memory (Barkley, 1997; Friedman & Miyake, 2004; Roberts & Pennington, 1996).

Assuming that arithmetic division accesses some form of cognitive arithmetic-fact network, previous research has suggested that suppression of inappropriate responses takes place to enable correct responses to predominate. Within this network, several responses are activated by a problem; suppression may take place in order to retrieve a correct response and this is a type of inhibition (Campbell & Clarke, 1989). Another theoretical concept suggested by Campbell and Clarke (1989) was that when solving a series of multiplication problems, a type of inhibition process takes place to filter previous recent responses which may be activated when solving the present problem. It does need to be borne in mind, however, that Campbell & Clarke (1989) examined multiplication only; in this study the time intervals between problems were varied (4.5s and 7.5s). Error and RT data suggested that inhibition may take time to initiate; this paper was pre-Friedman & Miyake (2004), hence the type of inhibition was not specified. There is neuroimaging evidence that similar areas of the brain that deals with activated multiplicative responses from a pair of numbers that are close together activate similar brain areas that are crucial for prepotent response inhibition (cf. Fulbright *et al,* 2003 and Swick, Ashley , & Turken, 2008). One might argue that a series of responses have the potential to become prepotent if they are identical, or distracters if they are not identical but, nevertheless, similar. If previous recent responses interfere, however, this suggests proactive interference rather than the other two types.

It has already been highlighted in Chapter Three, that there is consistent evidence that the central executive plays an important part in monitoring the carrying process and retrieval of responses from the long-term memory (Fürst & Hitch, 2000; Imbo, Vandierendonck & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2002). Whether one might be more specific in terms of attributing such functions to the two types of inhibition examined within the context of the present study is one of the aims of the experiment described in this chapter.

The first two experiments, in the present study, were aimed at examining the effects of prepotent response inhibition and resistance to distracter interference on the cognitive processes involved in solving complex division problems consisting of four-digit dividends and single digit divisors. Solving such problems is an extremely proceduralised process. The aim of Experiment Three was to break down the process involved in these division problems and focus on one procedure, namely, the 'short division' procedure and attempt to discover the effect on this of prepotent response inhibition and resistance to distracter interference. The problems that were used in Experiments One and Two were taken and fractionated into sub-problems, for example, 1456 ÷ 4 was split into three procedures: 14 ÷ 4, 25 ÷ 4 and 16 ÷ 4; the second procedure includes the digit 2 carried as a remainder from 14 ÷ 4 and the third procedure includes the digit 1 carried from 25 ÷ 4. As Experiment Three examined the short division process involved in these; participants were instructed to type the answers 3 for 14 ÷ 4, 6 for 25 ÷ 4 and 4 for 16 ÷ 4 and nothing else. The experimental conditions were the same as those covered in the first two experiments but the prepotent response inhibition and resistance to distracter interference conditions (cognitive conditions) were present in one experiment. In this way, all the arithmetic and both cognitive factors were varied entirely within-subjects, hence enabling all the effects to be investigated as a within-subjects study.

A limited amount of literature pertains to short division, some of which examines crystallised number-fact stores in LTM (Rickard, 2005; Rickard & Bourne, Jr., 1995, 1996) and whether or not division and multiplication share such stores (Campbell, 1997, 1999, LeFevre & Morris, 1999). Campbell (1997, 1999) used error-priming, i.e., priming a target trial with pre-trials from the same times-table with the intention of increasing the probability of participants responding with a close yet incorrect response to the target trial. For example, 4 x 7 might be preceded by 3 x 7, increasing the probability of responding to the latter problem with 28. LeFevre & Morris (1999) employed a self-report system, for one condition, where participants described the method they used for simple multiplication and division problems. Overall, the findings suggested memory for division facts was organised in terms of multiplicative relationships, hence, multiplication was used to mediate the division process, as the inverse operation, e.g., 12 ÷ 3 (= 4) is mediated by 3 (x 4) = 12 . It was also indicated that answers to small division problems (e.g., 10 ÷ 5; 6 ÷ 2) were retrieved directly from LTM (Campbell, 1999). Later work, however, suggests that both small and large division problems might be processed by either direct retrieval or inverse reference to multiplication facts depending on the skill level or preference of participants (Campbell & Alberts, 2010).

To summarise, the solving of multiple short-division problems may be mediated by multiplication and a set of responses within a cognitive 'multiplication chart' (Campbell, 1997, 1999; LeFevre & Morris, 1999; Rickard, 2005; Rickard & Bourne, Jr., 1996, 1996). Another possibility is that responses may be taken straight from a cognitive 'division chart' within the LTM (Campbell, 1999) or it may be dependent on participant preference (Campbell & Alberts, 2010). Either way, there may be competition from closely related or adjacent responses from within these division charts or Pythagorean tables (multiplication tables), for example, 7 x 6 may activate 49 and 36 as well as 42. In order for a correct response to be victorious, there needs to be an inhibition system to filter incorrect responses (Campbell & Clarke, 1989). Two types of inhibitory control are focussed upon, in the present study, employing an entirely within-subjects design in the following experiment.

From a theoretical perspective, the single-digit responses required in the present experiment are more likely to be taken direct from the LTM. One possible rôle for at least one type of inhibition mechanism may be to filter incorrect responses that are close to the correct ones, either from within a cognitive 'division-chart' or if a participant chooses to retrieve responses via mediation through multiplication, from a cognitive Pythagorean table. Secondly, participants would need to decide whether or not there is a need to carry a digit to the next column on the right. Moreover, a responsibility for at least one type of inhibition may be to aid the decision process with regard to whether or not a carry operation needs to be implemented. Experiment Three was designed to focus on the filtration or whatever procedure is involved in retrieving single digit responses with particular reference to PRI and RDI.

EXPERIMENT THREE

The following experiment was designed as a follow-up to Experiments One and Two. Experiment One investigated the effect of prepotent response inhibition on complex division, i.e., a four-digit dividend divided by a single-digit divisor (e.g., 1496 ÷ 4) under three arithmetic conditions: no carries, one carry and two carries; all problems were designed to yield a 3-digit response. Experiment Two examined the effect of resistance to distracter interference on the same problems. Owing to the extraction of the procedures from the more complex division process carried out by participants in the first two experiments, it was expected that the activities would become less cognitively taxing and hence result in relatively short latencies coupled with a lower error-rate. Inherent within the short division procedures and because of the nature of the problems with carrying, there was a new independent variable. A one-carry problem such as 1448 ÷ 4 will contain two divisible sub-problems: 24 ÷ 4 (= 6) and 8 ÷ 4 (= 2), these do not leave a remainder; and one non-divisible sub-problem, i.e., 14 ÷ 4 (= 3) which does leave a remainder. A two-carry problem such as 1496 ÷ 4 will have two non-divisible sub-problems, i.e., 14 ÷ 4 and 29 ÷ 4, coupled with one divisible sub-problem, i.e., 16 ÷ 4. To avoid confusion, the new independent variable containing divisible and non-divisible sub-conditions was designated ‘divisibility.’

The hypotheses were heavily based on the results of Experiments 1 and 2. It was predicted that Experiment 3 will result in an effect of prepotent response inhibition: increased RTs and error rates on short division, particularly for non-divisible sub-problems. In Experiment One, there was a significant increase in latencies when carrying was required, furthermore, there was an increase in error-rates when two carries were required; it follows that there ought to be elevated RTs and increased error-proneness, particularly regarding sub-problems that, if they were part of a complete problem, would trigger a carry. It was also predicted that resistance to distracter interference will result in a speed-accuracy trade-off (SAT) pattern in Experiment Three, i.e., error-rates will decrease as latencies increase. The results from Experiment Two suggested speed-accuracy trade-offs, particularly when one carry was required. Furthermore, if the type of inhibition referred to in Campbell & Clarke (1989) is indeed prepotent and the short division procedure is dependent on responses contained in a cognitive number-table then both latencies and error-rates should increase under the prepotent response inhibition load when compared with the control condition. The new independent variable led to the question: what effect, if any, will divisibility have on the latencies on error rates, hence, what will be its impact on the cognitive processing of division problems? Owing to this particular independent variable not being present during the first two experiments and there being no reference to it in the Literature, a speculative hypothesis was formulated that non-divisible problems would have elevated RTs and be more error prone as a result of PRI owing to greater load on the cognitive processing system; RDI, however would have the effect of reducing the error-rate.

The sub-problems in the present experiment are less proceduralised, in comparison to the problems used in Experiments One and Two. Moreover, for Experiment Three, both the cognitive and arithmetic conditions were varied entirely within-subjects. The cognitive factors were subjected to correlational analyses for each arithmetic condition to ascertain whether PRI and RDI worked together within the context of these less proceduralised tasks. It is hypothesised that PRI and RDI will correlate in terms of both RTs and error-rates.

The latter analyses of the previous chapter also highlighted the elevated latencies as a result of the flanker digits in comparison with those for PRI load under all three arithmetic conditions. Moreover, error-rates, as a result of exposure to flanker digits were considerably lower in comparison to those for PRI load whenever carrying was required. Extra analyses were carried out at the end of the Results section to explore whether these phenomena would be repeated when relatively short problems (sub-problems) were being solved. It was therefore predicted that there would be similar results for experiment 3 when PRI and RDI are compared resulting in slowed latencies under RDI when compared to PRI, regardless of the number of carries. Furthermore, it was predicted that RDI would result in a reduced error-rate, when compared to PRI, for problems requiring one and two carrying procedures.

Experiment Three was therefore designed to test the following hypotheses. (3.1) Based on the findings of Campbell & Clarke (1989) and from referring to the results of Experiment One, latencies will increase as a result of PRI when carrying is required. (3.2) From the results of Experiment Two, latencies will also increase as an effect of RDI regardless of the number of carrying operations. (3.3) From the results of Experiment One and the findings of Campbell & Clarke (1989), error rates will increase under the PRI condition when two carries are required. (3.4) From the results of Experiment Two, errors will reduce as a result of RDI when one carry is required which, when set against hypothesis 3.2, would be a speed-accuracy trade-off. Experiment Three was also tasked with attempting to answer the question regarding correlation between PRI and RDI, hypothesis (3.5), therefore was that RT and error means as a result of PRI and RDI would correlate. Hypothesis (3.6) was the prediction that RTs and error-proneness of non-divisible responses would have longer RTs and be more error-prone as a result of PRI and have elevated RTs but reduced error-proneness as a result of RDI. The extra analyses were designed to test the hypothesis (3.7) that RTs would be longer under RDI when compared to PRI, regardless of the number of carries. The hypothesis (3.8) that error-rates would be significantly alleviated under RDI when compared with PRI, under one and two carries, was also tested.

Method

Design

The overall experiment took the form of a 3 (0 vs. 1 vs. 2 carries – arithmetic factor) x 3 (Control vs. PRI vs. RDI - cognitive factor) design. The RTs for simultaneous secondary task (saying the direction of the arrows) were analysed separately in terms of four levels (control [direction] vs. 0 carries vs. 1 carry vs. 2 carries). Errors in the form of responding to the arrows in the PRI condition, and the control (direction) errors were analysed separately as non-parametric data). All conditions were varied entirely within-subjects; response times (in milliseconds) and error-rates were used as dependent measures. The cognitive conditions were counterbalanced within four groups of nine participants, in the following orders: 1) simultaneous → prepotent response inhibition → control (direction) → resistance to distracter interference → control; 2) resistance to distracter interference → control → simultaneous → prepotent response inhibition → control (direction); 3) control → resistance to distracter interference → simultaneous → prepotent response inhibition → control (direction); 4) simultaneous → prepotent response inhibition → control (direction) → control → resistance to distracter interference.

Participants

Thirty-nine participants originally volunteered who were recruited from the Schools of Health and Social Sciences, Games, Computing and Creative Technologies, Arts, Media and Education, Built Environment and Engineering, Bolton Business School and the Institute of Materials Research and Innovation at the University of Bolton. The data from three participants were discarded owing to excessive machine errors, leaving 36 participants whose data were processed. Sixteen participants were female and 20 were male; their approximate mean age was 30 years and ranged from 18 to 60; all were fluent in English and had normal or corrected to normal vision and none withdrew following the initial screening process.

Stimuli

All division problems were based on those comprising four-digit dividends between 1000 and 1999 and a single-digit divisor between 2 and 9. These problems were those from numbers 1 to 8 used in Experiments 1 and 2 (see Appendix III). However, for the purpose of this experiment, each problem was split into three sub-problems. Hence, a problem such as 1276/4 was split into 12/4, 7/4 and then 36/4. The third sub-problem was 36/4 because 7/4 has a remainder of 3; the 3 was automatically carried so that participants could concentrate on short division. Sub-problems were presented in the form of a top-heavy fraction (see Box 5.1).

1274/4 was displayed as:

$\frac{12}{4}$ followed by $\frac{7}{4}$ followed by $\frac{36}{4}$

Box 5.1. An Example of a Problem from the One-Carry Condition

For the simultaneous and inhibition conditions, each sub-problem was superimposed, within a white rectangle measuring approximately 75 x 30mm, onto a blue arrow measuring approximately 135mm between the vertical extremities and 280mm between the horizontal extremities. Each arrow was positioned on a white background to the left of the screen, horizontally but within the central area, vertically. The blue arrows pointed either to the left or right, at random. The arrows with superimposed problems were initially composed in PowerPoint, the numbers being in Calibri font, size 18; they were then transferred into Microsoft Paint, into Photo Gallery and then pasted into E-Prime objects. For the control condition (problems), the division problems were presented using black Courier New text, size 24 at the centre of the screen with no arrows present. For the control (direction) condition, the same sized arrows were used as in the control and inhibition conditions but with no problem superimposed on them; furthermore, they were repositioned slightly so they appeared at the centre of the screen.

There were 72 division problems in total, 24 different ones for each of three groups: group 1, for the simultaneous condition; group 2 for the prepotent inhibition condition and for the resistance to distracter interference conditions; and group 3, for the control condition (see Appendix III). Taking into account that each problem was split into three sub-problems, 72 responses were required in each cognitive condition, i.e., 288 single-digit responses for the main experiment. Within each group, 8 problems (24 sub-problems) were present forming three arithmetic conditions: no carries, one carry and two carries. Under the no-carry condition, a problem such as 1477/7 was split into 14/7, 7/7 and 7/7 whereas, under the one-carry condition, a problem, for example, 1322/2 was split into 13/2, 12/2 and 2/2. Finally, under the two-carry condition a problem such as 1572 was split into 15/6, 37/6 and 12/6.

Apparatus

The stimuli were presented on a Dell Desktop GX 280 computer coupled to a 43cm (17 inch) flat-screen colour monitor. A microphone was used to enable left / right vocal RTs to be collected via an E-Prime serial response box for the simultaneous and control (direction) conditions. E-Prime 2 (Schneider, Eschman & Zuccolotto, 2007) was installed on the equipment and used to control the experiment and collect data.

Within the settings for E-Prime 2, for the simultaneous condition, each arrow with superimposed problem was presented via a **pair** of ImageDisplay objects. Under the General properties of the first ImageDisplay object of the pair, this was aligned horizontally at 'left', aligned vertically at 'center' with the Clear After set at 'No.' Under Duration / Input, Duration was set at 0, the SRBOX was added along with the Allowable response, 6 and the Correct response, 6; the Time Limit was set at (infinite) and the End Action at (none); only the SR box RT was logged. The second ImageDisplay object was the same as the first with the following exceptions: under the general properties, Clear After was set at 'Yes,' under Duration / Input, the Duration was set at (infinite) and the Keyboard was added; the Allowable response option was set at {ANY} and 'Correct' was set to the correct response for the problem. The Time Limit was set at (same as duration) and the End Action was set at 'Terminate.' All other properties were set at their default values. No WaitObject was inserted in the middle of these pairs of ImageDisplay objects. In this way, the RTs for the vocal responses to the arrow directions and those for the typed answers to the problems could be collected simultaneously and no flicker could be detected as the computer moved from the first to the second ImageDisplay object of each pair. Within each group of problems, a 500ms blank screen (WaitObject) was inserted after each first and second ImageDisplay object and a 1250ms blank screen after each third, to signify the end of each problem.

For the prepotent response inhibition condition, each arrow with superimposed problem was presented via a single ImageDisplay object with the properties set as for the second of each pair under the simultaneous condition.

The problems for the resistance to distracter interference condition were originally typed in Microsoft PowerPoint (2007) in Calibri (Body) text, font size 24 and then copied into Microsoft

Paint, which automatically transferred each problem as a jpeg file in PhotoGallery. The PhotoGallery images were then programmed as ImageDisplayObjects into E-Prime 2.

Under the control condition each problem was presented using a TextDisplay object with the properties set at the same values as for the second of each pair of ImageDisplay objects under the simultaneous condition with the exception of AlignHorizontal under the General properties, which was set at 'center.' A 500ms blank screen was inserted between each TextDisplay object. For the Resistance to Distracter Interference condition, under General Properties, 'AlignHorizontal' was set at 'right,' AlignVertical at 'bottom.' Other settings were the same as for the control condition.

Under the control (direction) condition, each arrow was presented using an ImageDisplay object, each separated by a 500ms blank screen. With regard to the General properties, AlignVertical was set at 'top' and Clear After as 'Yes.' Under the Duration / Input properties, Duration was set at (infinite), the SRBOX was added to the devices; under Response Options, Allowable was set at 6, Correct at 6 and the Time Limit as (same as duration). Logging was requested for the RTs only.

Procedure

Participants indicated their age-range and gender on an information pro-forma to consent to take part in the exercise. The experimenter demonstrated the experiment by performing the necessary actions for three examples of problems under each of four conditions: simultaneous, prepotent response inhibition, resistance to distracter interference and control (problems) conditions, followed by three lone arrows under the control (direction) condition. This part of the procedure was repeated to enable participants to practise the exercise in the presence of the experimenter. It also acted as a screening device: the participant and the experimenter briefly discussed whether or not to continue with the main experiment based on the strength of the participant's accuracy when carrying out the division problems. Participants were instructed to press the [SPACE] bar to continue with the main part of the experiment after the experimenter had left the room. Throughout the main part of the experiment, the experimenter was present outside the room and viewed participants' progress through the window in the door; furthermore, oral responses to the arrow directions could be heard through the door and oral errors to arrow directions and false responses to 'prepotent response inhibition' condition requests to refrain from responding to arrow directions were recorded on paper by the experimenter.

For the first part of the simultaneous condition, participants initially responded to 24 division sub-problems, one at a time, from group 1 (see Appendix III) that were superimposed on a blue arrow that pointed either to the left or the right. They were instructed to say, loudly, which direction the arrow was pointing to and then type-in the single-digit integer answer to the problem – as quickly and as accurately as they could. The next problem did not follow a 500ms or 1250ms blank screen until one digit had been typed in on either the number keys above the letter keys or those on the numeric keypad. The problems were blocked under the no-carry arithmetic condition and began with three sub-problems with 2 as the divisor and then three more with 3 as the divisor and so on, consecutively until three sub-problems with 9 as the divisor were completed. After a set of on-screen instructions informed participants that the problems required one carry the same procedure was repeated under the one-carry arithmetic condition. A further set of instructions informed participants that the problems demanded two carries and the procedure was repeated under the two-carry condition.

At the end of the 72 problems (216 sub-problems), black text on a red background instructed participants to STOP saying, ‘left / right’ in response to the arrows and just answer the problems by typing. An instruction at the bottom of the screen asked them to press the [SPACE] bar when they were ready to continue.

Next, the first part of the prepotent inhibition condition, a further set of 72 problems (216 sub-problems) from group 2 (see Appendix III) were displayed, one at a time, still superimposed on a blue arrow. Participants were now expected to type-in the answers to the sub-problems, as before, but to refrain from saying, ‘left / right’ in response to the arrows and to press the [SPACE] bar when ready to continue. Any responses to the arrows were recorded as errors by the experimenter. The order of the problems and sub-problems was the same as under the simultaneous condition.

The control (direction) condition, consisted of a yellow instruction screen with black text asking participants to respond, vocally, ‘left / right’ to the following 24 arrows. These were presented one at a time on a white background with a 500ms blank screen in between each arrow; the computer moved to the next arrow as soon as a vocal response was detected. Any errors were recorded by the experimenter.

For the resistance to distracter interference condition, the problems were the same as those used under the prepotent inhibition condition; they were presented in the same order and in the same top-heavy fraction format. The sub-problems were presented with three contrasting digits on either side of the dividend and on either side of the divisor, e.g.,

88812888
8882888

The contrasting digits were different from all the digits within each problem (see Appendix I).

For the control condition (problems), a set of instructions on a white background then required participants to type-in answers only to the following problems. Twenty-four problems (72 sub-problems) from group 3 were then presented in the same blocked order, split into sub-problems, as described under previous cognitive conditions, in the following format:

16
4

Results

Response Times

The Simultaneous and control (direction) conditions were analysed separately from the rest of the experiment as these were designed as a manipulative activity to induce a prepotent response, and as a comparative condition with the arrow-response in the simultaneous condition, respectively; these can be found in Appendix IV and will not form part of this results section. Hence, for the initial part of the analysis, the simultaneous condition responses were eliminated: the simultaneous condition was designed as a conditioning activity to induce a prepotent response to be inhibited under the prepotent inhibition condition; furthermore, the simultaneous condition probably tapped input monitoring, response selection and the phonological loop. Only the latencies of the correctly solved problems coupled with correct responses to the direction of the arrows (in the case of the simultaneous condition) were analysed.

Prior to analysis, the data were examined and machine errors, owing to coughs and the failure of the microphone to record responses, were removed from the analysis; 8.45% of the data were eliminated in this way. The remaining data were screened participant-by-participant; any data ±2 standard deviations away from the mean were replaced by the mean; this accounted for 4.22% of the data.

Table 5.1

Mean RTs (ms): Control, Prepotent Response Inhibition and Resistance to Distracter Interference (N = 36)

Condition		Mean	SD
Control	*No Carries*	1183	273
	One carry	1804	477
	Two Carries	2488	734
Prepotent Response Inhibition	*No Carries*	1245	310
	One carry	1762	610
	Two Carries	2488	947
Resistance to Distracter Interference	*No Carries*	1314	330
	One carry	1912	618
	Two Carries	2883	945

A 3(control vs. PRI vs. RDI) x 3(0 vs. 1 vs. 2 carries) repeated measures ANOVA revealed a significant main effect of cognitive factor (Control, PRI and RDI), $F(2, 70) = 10.06$, $p < 0.001$, $\eta^2_p = 0.22$ and a significant main effect of arithmetic factor (0, 1 and 2 carries), $F(2, 70) = 151.95$, $p < 0.001$, $\eta^2_p = 0.81$. There was also a significant cognitive factor x arithmetic factor interaction, $F(4, 140) = 3.87$, $p = 0.005$, $\eta^2_p = 0.10$.

A series of *post hoc* tests with stepwise partial Bonferroni correction were carried out and revealed two significant increases in RTs: Control vs. RDI (two carries), $t(35) = -5.18$, $p < 0.001$, $\alpha = 0.0056$ and Control vs. RDI (no carries), $t(35) = -3.80$, $p = 0.0005$, $\alpha = 0.011$. No other comparisons were significant, $p > \alpha$. As only significant increases or significant decreases in latencies were of interest, all tests were one-tailed. These two significant increases in latencies as an effect of RDI coupled with the statistically non-significant comparisons with respect to the remaining conditions were deemed the cause of the interaction.

These results did not support hypothesis 3.1: PRI appears to have had no effect on latencies with respect to any arithmetic condition and therefore appears to cause little or no interference. They did, however, partially support hypothesis 3.2 in that RDI interfered with the cognitive short-division process when no carries and when two carries were required – but not when one carry was implemented. These represent the least difficult and most difficult problems.

As suggested in the introduction to this chapter, it was apparent that further arithmetic process-related conditions existed. These were generated from two of the arithmetic conditions: one and two carries. Owing to this, and before assuming that PRI has no effect on latencies whatsoever, it was considered beneficial to study these two (new) conditions more closely. As a reminder, under the one-carry condition, for example, a series of three short division procedures based on a problem similar to 1416/ 3 were undertaken by participants, one procedure required carrying (14/3) and two did not (21/3 and 6/3). Whereas, under the two carries condition, e.g., 1451/3, two procedures demanded carrying (14/3 and 25/3) and one did not (21/3). To avoid confusion, these secondary arithmetic conditions were defined as 'divisible' (e.g., 21/3 divides without leaving a remainder) and 'non-divisible' (e.g., 14/3 divides but leaves a remainder of 2). Under the no carry arithmetic condition, all problems contained divisible procedures; hence, for the purpose of further analysis, this condition was dispensed with. Table 5.2 shows the new mean latencies as a result of fragmentation following the inclusion of these arithmetic sub-conditions.

Table 5.2.

Mean RTs: Sub-conditions Included (N =36)

Condition			Mean RT (ms)	SD
Control	One Carry	*Divisible*	1590	412
		Non-Divisible	2227	702
	Two Carries	*Divisible*	1947	642
		Non-Divisible	2758	807
Prepotent Response Inhibition	One Carry	*Divisible*	1590	578
		Non-Divisible	2088	731
	Two Carries	*Divisible*	2398	1045
		Non-Divisible	2792	1004
Resistance to Distracter Interference	One Carry	*Divisible*	1762	590
		Non-Divisible	2216	741
	Two Carries	*Divisible*	2880	1134
		Non-Divisible	2879	916

Prepotent Response Inhibition (PRI) and Resistance to Distracter Interference (RDI) were examined separately by implementing two 2 (Control vs. Inhibition) x 2 (1 carry vs. 2 carries) x 2 divisible vs. non-divisible) ANOVAs. These were followed up by a series of eight one-tailed *post hoc* tests comparing the means of the RTs between control and PRI and control and RDI conditions under one-carry, two carries coupled with their divisible and non-divisible variants; no correction would have made any differences to the outcomes. Firstly, the ANOVA on Control vs. PRI indicated a significant main effect of arithmetic factor, $F(1,35) = 91.13$, $p < 0.001$, $\eta^2_p = 0.72$, reflecting increased cognitive load as problem difficulty intensified. There was a significant main effect of divisibility, $F(1,35) = 151.18$, $p < 0.001$, $\eta^2_p = 0.81$, reflecting elevated cognitive load for non-divisible sub-problems. There was no significant main effect of PRI, $F = 1.52$, $p > 0.05$. A significant cognitive factor x arithmetic factor interaction was reported, $F(1,35) = 13.89$, $p = 0.001$, $\eta^2_p = 0.28$ and also a significant cognitive factor x divisibility interaction, $F(1,35) = 9.75$, $p = 0.004$, $\eta^2_p = 0.22$. No significant arithmetic factor x divisibility interaction was reported, $F = 0.15$, $p > 0.05$ but there was a small but significant three-way interaction $F(1,35) = 4.51$, $p = 0.04$, $\eta^2_p = 0.11$, reflecting the trend towards a larger difference between latencies regarding sub-problems that were divisible under two-carries as a result of PRI load, in opposition to the relatively insignificant differences caused by PRI load on the other types of sub-problems. The series of *post hoc* tests revealed a significant increase form control to PRI load where the procedure was divisible under the two-carries condition, $t(35) = -3.95$, $p < 0.001$. These results represent partial support for hypothesis 3.1 that RTs would increase as a consequence of PRI load but hypothesis 6 predicted an increase in RTs for non-divisible sub-problems; it did not predict such an elevation in latencies for *divisible* sub-problems, as reported here (see Figures, 5.1a and 5.1b).

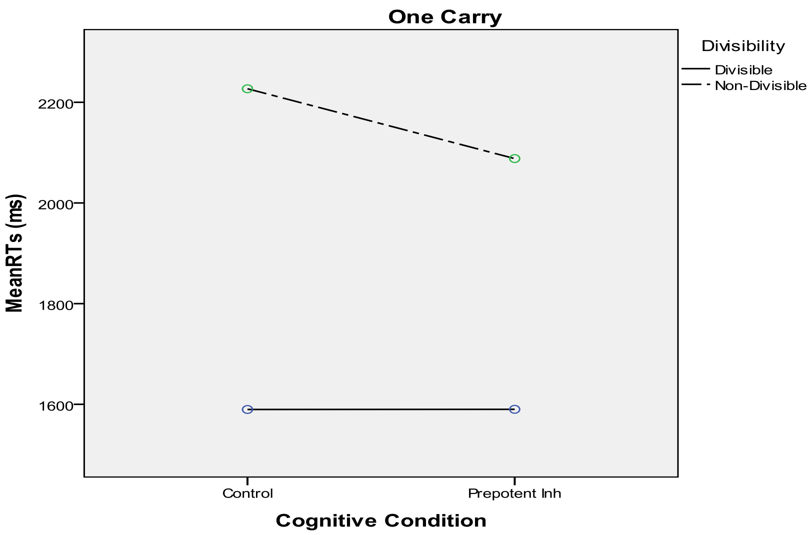

Figure 5.1a. Mean RTs: PRI Effects on Divisible / Non-Divisible Conditions (One Carry)

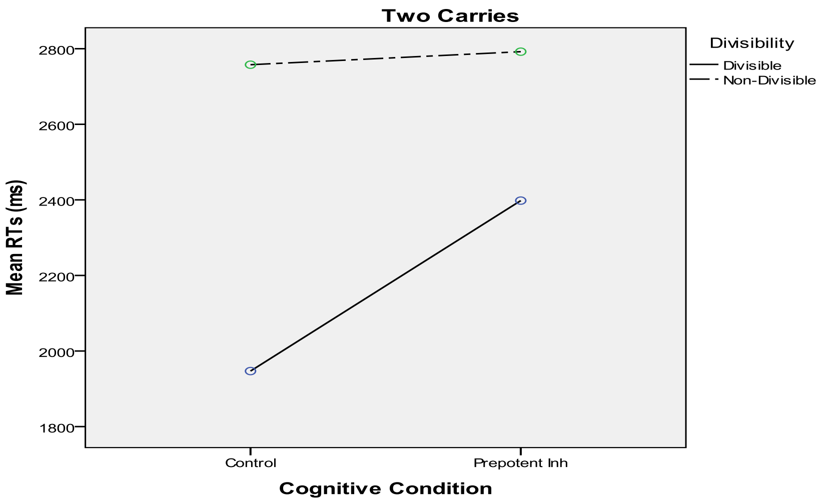

Figure 5.1b. Mean RTs: PRI Effects on Divisible / Non-Divisible Conditions (Two Carries)

Secondly, the ANOVA on Control *versus* RDI revealed a significant main effect of RDI, $F(1,35) = 28.60$, $p < 0.001$, $\eta^2_p = 0.45$, reflecting the cognitive interference induced by RDI. There was also a significant main effect of arithmetic-factor, $F(1,35) = 102.79$, $p < 0.001$, $\eta^2_p = 0.75$, and divisibility, $F(1,35) = 98.43$, $p < 0.001$, $\eta^2_p = 0.74$. These reflected the increases in cognitive load as a result of the number of carries and non-divisibility. It furthermore disclosed three significant interactions: cognitive-factor x arithmetic factor, $F(1,35) = 21.34$, $p < 0.001$, $\eta^2_p = 0.38$; cognitive factor x divisibility, $F(1,35) = 37.92$, $p < 0.001$, $\eta^2_p = 0.52$; three-way, $F(1,35) = 16.13$, $p < 0.001$, $\eta^2_p = 0.32$, reflecting the increased slowing of latencies with respect to divisible sub-problems under two carries, in comparison to the one carry and non-divisible conditions. There was no significant arithmetic factor x divisibility interaction, $F = 3.25$, $p > 0.05$, see Figures 5.2a and 5.2b.

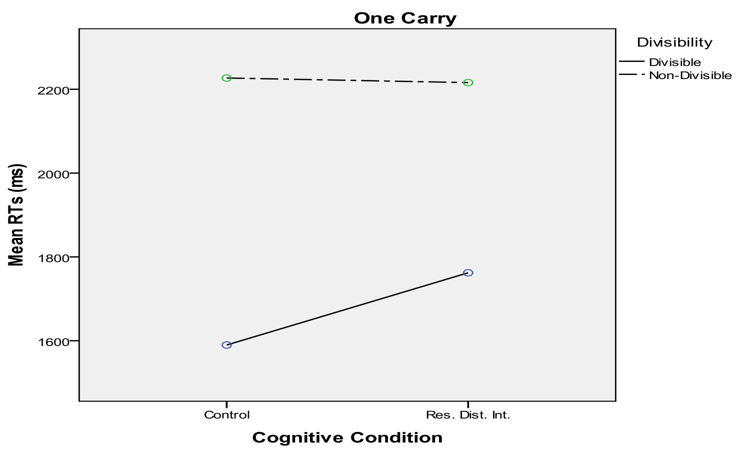

Figure 5.2a. Mean RTs: RDI Effects on Divisible / Non-Divisible Conditions (One Carry)

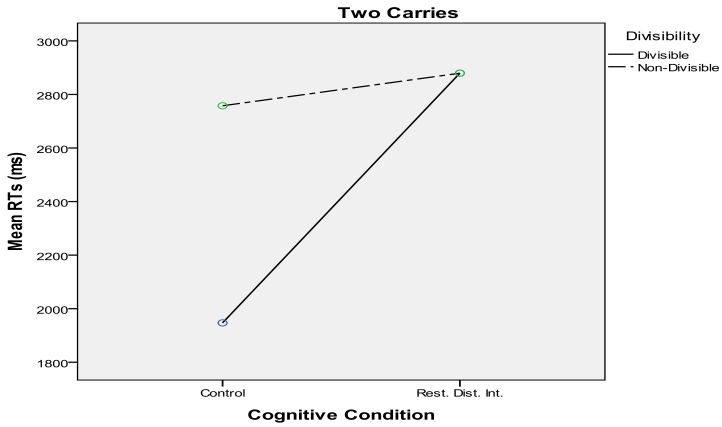

Figure 5.2b. Mean RTs: RDI Effects on Divisible / Non-Divisible Conditions (Two Carries)

The series of *post hoc* tests revealed a significant increase from control to resistance to distracter interference, again, where the procedure was divisible under two-carries, $t(35) = -6.99$, $p < 0.001$. None of the other *post hoc* tests revealed any significant comparisons, $p > 0.05$. Re-examining the hypotheses, where two carry operations were required and the sub-problems were divisible, then Hypothesis 3.2, that RDI would cause a significant increase in latencies was partially supported: RDI had a similar effect on the same type of problem as PRI but more intensively. Hypothesis 3.6 was not supported in that RDI affected divisible problems rather than those that were non-divisible. The one-carry condition was not significantly affected, in terms of divisibility, as a result of either type of inhibition.

Errors

Table 5.3

Mean Percentage Error Rates: Control, PRI and RDI (N = 36)

Condition		Mean Errors (%)	SD
Control	*No Carries*	2.66	3.47
	One carry	6.60	6.93
	Two Carries	11.46	12.53
Prepotent	*No Carries*	3.01	3.94
Response	*One carry*	5.21	5.49
Inhibition	*Two Carries*	9.72	9.24
Resistance to	*No Carries*	1.62	2.69
Distracter	*One carry*	4.86	7.01
Interference	*Two Carries*	12.73	11.99

Consistent with the analysis of latencies, a 3 (Control vs. PRI vs. RDI) x 3 (0 vs. 1vs. 2 carries) repeated measures ANOVA was carried out on the percentage error-rates, revealing a significant main effect of arithmetic factor (no carries, one carry, two carries), $F(2, 70) = 32.25$, $p < 0.001$, $\eta^2_p = 0.48$, reflecting increased cognitive load as the problem difficulty intensified, which is only to be expected. There was no significant main effect of cognitive factor, $F = 1.13$, $p > 0.05$. There was a significant but small cognitive-factor x arithmetic-factor interaction, $F(4, 140) = 2.67$, $p = 0.035$, $\eta^2_p = 0.07$, reflecting the slightly differing trends (some increases and some reductions) in error-rates depending on PRI or RDI, see Table 5.3.

A series of *post hoc* tests with partial Bonferroni correction revealed no significant comparisons: control *versus* prepotent response inhibition and control *versus* resistance to distracter interference under all three arithmetic conditions, $p > \alpha$, suggesting that the interaction was not of great importance. Moreover, these results did not support hypotheses 3.3 and 3.4 in that there were no significant increases in error-rates under PRI and no significant decreases under RDI (see table 5.4). The error data were again separated into divisible and non-divisible sub-conditions to maintain consistency with the analyses on latencies and to probe possible differing effects as a result of PRI and RDI on these sub-conditions.

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Table 5.4

Mean Error Rates (%): Sub-conditions Included (N =36)

Condition			M	SD
Control	One Carry	*Divisible*	5.73	7.53
		Non-Divisible	8.33	12.32
	Two Carries	*Divisible*	10.07	11.50
		Non-Divisible	12.15	15.37
Prepotent Response Inhibition	One Carry	*Divisible*	6.08	5.49
		Non-Divisible	3.47	10.18
	Two Carries	*Divisible*	12.15	12.13
		Non-Divisible	8.51	10.04
Resistance to Distracter Interference	One Carry	*Divisible*	4.34	5.94
		Non-Divisible	5.90	12.50
	Two Carries	*Divisible*	15.97	16.80
		Non-Divisible	11.11	11.68

Two 2 (Control vs. PRI) x 2 (One carry vs. Two carries) x 2 (divisible vs non-divisible) repeated measures ANOVAs were undertaken. Firstly, with regard to control *versus* prepotent response inhibition, there was a significant main effect of arithmetic-factor, $F(1,35) = 18.41$, $p < 0.001$, $\eta^2_p = 0.35$, reflecting the expected increased error-proneness when two carries took place, in comparison to one-carry. No significant main effects of cognitive-factor or divisibility were evident, $F = 2.12$, $p > 0.05$ and $F = 0.1$, $p > 0.5$, respectively. One significant interaction was evident: cognitive-factor x divisibility, $F(1,35) = 5.59$, $p = 0.024$, $\eta^2_p = 0.14$, reflecting the trend towards lower error-rates for non-divisible problems under PRI, in comparison with the control condition, see Table 5.4. None of the other interactions were significant.

Secondly, in the case of control versus resistance to distracter interference, there was one significant main effect: that of arithmetic factor, $F(1,35) = 29.91$, $p < 0.001$, $\eta^2_p = 0.43$, indicating the increased error-proneness of two carry problems. Two significant interactions were disclosed: cognitive factor x arithmetic factor, $F(1,35) = 5.10$, $p = 0.03$, $\eta^2_p = 0.13$, indicating a trend towards a lower error-rate as a result of RDI under the one-carry condition. There was an arithmetic factor x divisibility interaction, $F(1,35) = 4.50$, $p = 0.041$, $\eta^2_p = 0.11$, reflecting the trend towards a lower error-rate for non-divisible two-carry problems under RDI. There was no significant three-way interaction. The trends towards lower error-rates for non-divisible problems did not support hypothesis 3.6 which predicted a general rise in error-proneness for non-divisible problems.

A series of one-tailed t-tests with partial Bonferroni correction revealed no significant differences with any pair-wise comparison, control versus prepotent response inhibition and control versus resistance to distracter interference, under the one and two carries arithmetic conditions, $p > \alpha$, indicating a lack of importance, possibly owing to the size of the effects, with regard to the interactions. These results did not support hypotheses 3.3 or 3.4 in that any increase or decrease in error rates could not be attributed to either prepotent response inhibition or resistance to distracter interference. They could, however be attributed to arithmetic condition, but this is only to be expected: carrying increases problem-difficulty and hence error-proneness.

Comparison between Prepotent Response Inhibition and Resistance to Distracter Interference in Terms of the Short Division Process

The two induced cognitive conditions were compared separately to ascertain whether a significant difference in the distribution of latencies and error-rates existed between them. The latencies were compared first, to test hypothesis 3.7 that predicted slower latencies for RDI when compared to those for PRI.

A series of three one tailed paired sample t-tests, with correction, on the data before breaking down the procedures for divisibility revealed one significant difference: an increase in mean RTs, from prepotent response inhibition to resistance to distracter interference, when two carries where required, $t(35) = -2.07$, $p = 0.023$, $\alpha = 0.034$, partially supporting hypothesis 3.7. The overall directions of the changes in RTs, however, from condition to condition generally tended to flow in a similar direction with overall RTs being slower under the RDI condition. Pearson's two-tailed correlations were therefore calculated on the latencies both before and after divisibility were separated from each arithmetic condition. These indicated significant correlations between the data in the same arithmetic condition under the two types of inhibition. All correlations comprised PRI vs. RDI and the r values were as follows: no carries, $r = 0.64$, $p < 0.01$; one carry, $r = 0.76$, $p < 0.01$; two carries, $r = 0.79$, $p < 0.01$, supporting hypothesis 3.5 that the latencies from PRI load and RDI would correlate.

Decomposition of the data was implemented in order to take divisibility into account. A series of four paired-samples t- tests revealed two significant differences: an increase in mean RTs, from PRI to RDI under 'one-carry' (divisible), $t(35) = -2.43$, $p = 0.01$, $\alpha = 0.025$, and the same but under 'two-carries' (divisible), $t(35) = -3.83$, $p = 0.0005$, $\alpha = 0.0125$, representing partial support for hypothesis 3.7 that RTs would be slower for the RDI condition. This might suggest a slightly more substantial rôle for resistance to distracter interference in the decision-making process: 'to carry, or not to carry. Pearson's two-tailed correlations indicated significant correlations between the data in the same arithmetic conditions and divisibility ratings under the opposing cognitive conditions: PRI vs. RDI (1 carry/ divisible), $r = 0.74$, $p < 0.01$; PRI vs. RDI (1 carry,/ non-divisible, $r = 0.73$, $p < 0.01$; PRI vs. RDI (2 carries/ divisible), $r = 0.76$, $p < 0.01$; PRI vs. RDI (2 carries/ non-divisible), $r = 0.72$, $p < 0.01$, providing further support for hypothesis 3.5.

From the Pearson's correlations and results from the paired samples tests, it follows that RDI slows response times significantly more during the processing of problems requiring either one or two carries and when the procedures contain a divisible dividend. This was a new phenomenon which was not predicted and came to light as a result of separating divisibility as another independent variable; it was also an aspect that was similarly noted following the t-tests after the initial ANOVA on the RTs.

The error data were compared using similar analyses. This time, a series of three one tailed corrected paired-sample t-tests on the data prior to decomposing the procedures for divisibility revealed two significant differences: an increase in mean error-rates, from PRI to RDI, when two carries where required, $t(35) = -2.09$, $p = 0.014$, $\alpha = 0.017$; and a *decrease* when no carries were needed, $t(35) = 2.16$, $p = 0.019$, $\alpha = 0.034$. The results for two carries did not support hypothesis 3.8 that predicted a reduction in error-rates under RDI, but the results for no carries did. These opposing results are consistent with the cognitive factor x arithmetic factor interaction reported in the initial ANOVA on error-rates (*c.f.*, Table 5.3).

Pearson's correlations indicated significant correlations between the mean error-rates in the same arithmetic conditions when comparing PRI with RDI: PRI vs. RDI (no-carries), $r = 0.37$, $p < 0.05$; PRI vs. RDI (one carry), $r = 0.73$, $p < 0.01$; PRI vs. RDI (two-carries), $r = 0.75$, $p = 0.01$. The correlations supported hypothesis 5 that predicted that the two types of inhibition would correlate. The lower strength of the correlation for no carries indicates that PRI and RDI work together more strongly when carrying is required.

When the data were decomposed to take into account divisibility, a series of four corrected paired-sample t-tests tests revealed no significant differences, $p > \alpha$, reflecting weakened differences when the arithmetic procedures were further separated. This did not support hypothesis 3.8 that errors would be fewer under RDI. Significant Pearson's correlations were apparent between the data in parallel arithmetic conditions and divisibility ratings for PRI vs. RDI: 1 carry (divisible), r = 0.40, $p < 0.05$; 1 carry (non-divisible), $r = 0.85$, $p < 0.01$; 2 carries (divisible), $r = 0.64$, $p < 0.01$; 2 carries (non-divisible), $r = 0.52$, $p < 0.01$. These r values supported hypothesis 3.5 that the mean error-rates from PRI and RDI would correlate; there was a notable reduction in correlational strength where the procedures required one carry and had divisible dividends.

Individual Differences

To ascertain whether or not any effects were as a result of differing arithmetic abilities amongst participants, a group of thirteen participants were selected based on the criterion, one error or fewer in each arithmetic condition, no carries, one, and two carries within the 'control' cognitive condition; their data formed the 'stronger' variable A MANCOVA was undertaken and the stronger variable was entered as the covariate (under control, prepotent inhibition and resistance to distracter interference and all three arithmetic conditions) and the data from the remaining twenty-three were entered as the dependent variable. This analysis was carried out for both accuracy and latencies. None of the group comparisons between equivalent factors indicated a significant difference, $p > 0.05$.

Discussion

The aim of Experiment Three was to investigate the effect of prepotent response inhibition and resistance to distracter interference on the short-division procedure within the complex division process. The effects of the direction-stating activity were considered less important for the purpose of this experiment owing to the lack of purity because the activity was designed as a conditioning or manipulation task, i.e., to induce a prepotent response in participants that subsequently had to be inhibited. The results for this manipulation activity do suggest that it served its purpose and are provided in Appendix IV.

Of greater importance were the effects of the inhibition-conditions on the short division process – these being part of the procedure used by educated adults to solve complex division problems (e.g., 1245 ÷ 5). Hypothesis 3.1 stated that latencies would increase as a result of PRI. This was partially supported but only in a small way. There was a significant increase in RTs as a result of PRI; however, it did depend upon the sub-problems being *divisible* and only under two carries. Discussing PRI first, the findings with regard to RTs that were significant were discovered when the latencies were separated into those from divisible and non-divisible sub-problems, under the one and two-carries conditions. There were increased latencies as a result of PRI under the two-carry condition when, and only when the sub-problems had divisible dividends. It needs to be borne in mind that, under the two-carry condition, there were half the number of divisible dividends as those non-divisible. Moreover, these had a regular rhythmical pattern: non-divisible, non-divisible, *divisible*; non-divisible, non-divisible, *divisible,* and so on. Where dividends are divisible (e.g., 16/4), responses will be exact; whereas, where dividends are non-divisible (e.g., 17/3), responses will be approximate. The nature of this main effect of prepotent response inhibition might therefore suggest that, within the context of the present experiment, it may have a rôle in inhibiting approximate responses where exact answers are required. A further explanation lies inherently within the design of the present experiment. When two carries were required, sub-problems with approximate responses may be deemed the prepotent responses, there not only being twice as many approximate responses, but the nature of the two carry problems demands two approximations followed by one exact response. It is plausible that, for it to be correct, every third response could be aided by a mechanism to *inhibit* an approximate response.

Hypothesis 3.2, which stated that latencies would be slowed owing to the effect of RDI, was partially supported in that the RTs were slowed when problems demanded no carries and two carries but not one carry. The hypothesis was further supported, at least, partially: RDI had a similar effect to PRI on the same type of problem (two-carries) but with a prerequisite upon the sub-problems being *divisible*. Obviously, problems under the no-carry condition all comprised sub-problems that were divisible and from this, it followed that all correct responses were *not* approximations. There may be more than one explanation for the effect of RDI. It might suggest possible conflict involving approximate and exact responses, the approximate responses being the external interference. Another explanation might be that, parallel to the notion suggested in the discussion of Experiment Two that RDI was a cognitive mechanism that was triggered rather than loaded, resulting in a general slowing owing to more focussed attention on the tasks. As already stated, the two carry condition contained a series of '2 non-divisible responses followed by 1 divisible response.' The effect on complete problems in Experiment Two was similar to that for no carries but, according to the *t* value, statistically more intense. More focussed attention on the two-carry problems (that contained approximate-response /exact-response interference) would obviously be beneficial for accuracy – this would partially explain the slowed RTs when RDI was implemented. Furthermore, there were increased latencies as a result of RDI, similarly to PRI, under the two-carry condition when, and only when the sub-problems had divisible dividends. This could be interpreted as a case of PRI and RDI working together to create a mechanism to *inhibit* approximate responses, possibly with RDI treating these as external interference and PRI treating them as prepotent responses after having detected two consecutive such responses. Having forwarded such a notion, however, two consecutive non-divisible sub-problems seem rather few in number to create a prepotent response-state and should possibly be treated with a certain amount of caution.

Another possible and rather tentative explanation is that exact responses demand more working memory resources than approximate responses. This is tentative in that the evidence for this is taken from an investigation into double-digit addition (Kalaman & LeFevre, 2007). It was furthermore suggested that the main causal factor in this increase in working memory demand was the carrying operation in the more demanding double-digit additions. This notion may be further supported by evidence that rounding down demands fewer working memory resources than rounding up (Lemaire & Lecacheur, 2002). The sub-problems in the present experiment all demanded integer responses, hence they had to be rounded down (e.g., $18 \div 5 = 3$, rather than 4, which would be the normal approximation). The latter explanations are less feasible, within the context of the present study: the studies quoted involved the study of addition rather than division; moreover, the approximations for the present experiment did not demand any carrying, as such; carrying was programmed into the design of the computer activities so that participants did not have to do it. Participants were instructed to enter single digit integer responses only, with the intention of reducing the arithmetic load on the cognitive system to a minimum so that short division procedures could be isolated.

Regarding hypotheses 3.3 and 3.4 which stated that error-rates would increase both as an effect of PRI and of RDI, both these were refuted and any significant interaction rendered unimportant by the *post-hoc* investigations. The results of Experiment One revealed increased errors as an effect of PRI under the two-carry arithmetic condition and those of Experiment 2 revealed some reductions in errors as an effect of RDI, under the no-carry and one-carry arithmetic conditions. The overall inference would appear to be that both PRI and RDI have a rôle in monitoring errors over the whole process of solving a division problem but not within the context of the extracted short-division procedures. It was proposed in the discussion for Chapter Four that RDI is sensitive to problem difficulty and has the ability to intensify, as and when required.

Moreover, also refuted was the hypothesis that RDI would cause a speed-accuracy trade-off (hypothesis 3.4, in conjunction with hypothesis 3.2). Campbell & Clarke (1989) proposed that responses were drawn from a cognitive network of number-facts (tables-chart), the closer ones to the correct response of which need to be inhibited. If such inhibition was a rôle for PRI and / or RDI, then one might have expected an effect of these on error-rates. As no such effect revealed itself, their responsibility seems more akin to monitoring procedures rather than numerical accuracy, the most obvious procedure being carrying itself (cf. Fürst & Hitch, 2000). Investigation into carrying is the prerogative of the following chapter.

The correlational analyses comparing PRI with RDI yielded significant positive results. Hypothesis 3.5 predicted that PRI and RDI would correlate in terms of both latencies and error-rates. Such correlations were consistent with the notion (Friedman & Miyake, 2004) that PRI and RDI work closely together.

The paired samples comparisons between PRI and RDI suggested that RDI had a far greater slowing effect on latencies and a greater interference effect in terms of increasing errors and was consistent with prediction (hypothesis 3.7). There appears to be an indication that the effects of the two types of inhibition are similar on the short division procedure but RDI has a more intense effect.

These results were indicative of a differing rôle for the two types of inhibition depending on whether the task comprises a single procedure as with Experiment Three, or a multi-proceduralised process that was expected in Experiments One and Two. The differences in latencies between PRI and RDI further support the notion expressed in the discussion for Experiment Two that RDI, which caused longer RTs, was *triggered* as soon as the problems hidden amongst the flanker digits were seen. Owing to this triggering, RDI takes on a proactive involvement which is constantly monitoring the difficulty of the problems and intensifies when problem difficulty intensifies. On the other hand, PRI (faster RTs than RDI) is a type of inhibition that either presents itself later or enters from time to time, as and when required, in order to filter some form of prepotent response. The initial decade of the present century has seen research into mechanisms of cognitive control such as conflict monitoring theory (Botvinick, Braver, Barch, Carter & Cohen, 2001) and dual-mechanism of control theory (Braver, Gray & Burgess, 2007). Conflict monitoring theory suggests that response conflict triggers conflict adaption in that the conflict monitoring system assesses the intensity of the conflict and transfers this information to the cognitive centres responsible for control; consequently, these centres adjust the strength of their influence on the cognitive processing system (Botvinick *et al* 2001). Dual mechanism of control theory suggests that cognitive control operates via two operating modes that are dissociable: proactive control and reactive control (Braver *et al*, 2007). Proactive control has a rôle in anticipating and preventing interference whereas reactive control is involved in the detection and resolution of interference after it has started (Braver, Paxton, Locke & Barch, 2009). It might be argued that RDI has a responsibility towards proactive control and PRI is more of a reactive process. The results of the three experiments so far, appear to proceed, at least some way, towards supporting such theories.

In conclusion, and synthesising what has been discussed so far, one might argue that prepotent response inhibition is part of a reactive control mechanism that monitors breakages of prepotent response expectations (such as the '2 non-divisible responses followed by 1 divisible response' pattern) so that, where necessary, the unwanted prepotent response-type is inhibited in favour of a correct response-type. Resistance to distracter interference, on the other hand, may be looked upon as part of a proactive control mechanism that detects possible conflict (e.g., flankers) and such conflict triggers it into constant monitoring of response-types in order to filter the unwanted ones. The term response-type is used here to separate it from 'response' which might, on its own, imply a definitive numerical value; non-divisible response-types may be regarded as approximate answers whereas divisible response-types are exact.

More will be discussed about the relationship between the two types of inhibition within this study and the above theories in the overall discussion. Meanwhile another important procedure within the process of complex division is the carrying process; the following experiment was designed to examine this.

CHAPTER SIX

The Effect of Prepotent Response Inhibition and Resistance to Distracter Interference on the Carrying Processes within the Complex Division Procedure

The previous chapter examined the effect of prepotent response inhibition and resistance to distracter interference on the short division processes. The results suggested that prepotent response inhibition (PRI) and resistance to distracter interference (RDI) may each be part of a cognitive monitoring process in which PRI forms part of a *reactive* control mechanism that activates, as and when necessary, to filter unwanted prepotent response patterns in favour of correct ones. With respect to RDI, this may be looked upon as part of a *proactive* control mechanism that detects possible conflict at the outset (in the case of the present study, flanker-digits). Such conflict may trigger RDI into constant monitoring of response-types to enable the filtering of unwanted ones (*cf.*, Botvinick *et al*, 2001, Braver *et al*, 2007 & Braver *et al*, 2009). Another interesting set of results were the strong correlations between the two types of inhibition in terms of both RTs and error-rates. This suggested behavioural experimental support for the notion forwarded by Friedman & Miyake (2004) that PRI and RDI were closely related.

Experiment Four represents an attempt to extract the 'carrying' processes from the complex division procedure and study the effect of the two types of inhibition on this part of the whole division procedure, using the same sub-problems as those used for Experiment Three and also similar analytical *modus operandi*. To reduce the risk of interference from the short division process, participants were provided with the 'short division response' and asked to type the remainder, thus:

$$\frac{17}{5} = 3 \quad r? \quad \text{Participants were expected to type, 2.}$$

Carrying in arithmetic division, according to standard algorithms may be implemented in at least two ways. Suppose we have a division problem such as 1524 ÷ 4, it could be proceduralised as follows:

-

$$
\begin{array}{r}
\underline{381} \\
4\ )\ 1524 \\
\underline{12} \\
32 \\
\underline{32} \\
04
\end{array}
$$

Here: four into 15 enters 3 times, 15 – 12 = 3, 2 is brought down from the decades column to form 32; four into 32 enters 8 times, 32 – 32 = 0, 4 is brought down from the units column; four into four enters 1 time, hence the answer is 381. On the other hand, when the divisor is a single digit, an alternative procedure is:

$$
\begin{array}{r}
\underline{3\,81} \\
4\ )\ 15^{3}24
\end{array}
$$

Four enters into fifteen 3 times, 15 - 12 = 3, so *carry* the 3 and place it in front of the 2 to form 32; four into 32 enters 8 times, as 8 x 4 = 32, there is no remainder, hence four into 4 enters 1 time, therefore the answer is 381. This latter procedure is what participants were encouraged to carry out during the first two experiments. Some controversy is evident with regard to how division is taught in schools today and there tends to be an emphasis on referring back to the inverse operation of multiplication, i.e., if 9 x 5 = 45, it follows that 45 ÷ 9 = 5 and present day school pupils may be encouraged to formulate their own algorithms (which may or may not be generalisable to problems of the same type). However, the previous two suggested methods are standard algorithms that can be used for all problems of the same type (Klein & Milgram, ND); for this reason, it was the latter method that participants were encouraged to use in the present study.

Apart from the notions that related inhibition to the possible prepotent tendency ‘not to carry’ (Fürst & Hitch, 2000; Imbo, Vandierendonck, & Vergauwe, 2007), and also the possibility that when solving a series of problems that involve carrying, previous responses might interfere with the problem being solved immediately before (Imbo, Vandierendonck, & Vergauwe, 2007), little has been written about the cognitive processes involved in carrying, as a specific procedure. Early research has investigated procedural aspects of carrying from a mathematical perspective, but not necessarily within the context of division. Widaman and associates investigated a number of processes in mental addition, including retrieval of addition facts and carrying. Four arithmetic conditions were presented in this study: a single digit number added to a single digit number, a double digit addend coupled with a single digit addend, three single digit addends, and, more pertinent to the present study, two double-digit numbers added together – these demanded carrying. Little was inferred about carrying, apart from the mean processing speed of the carrying procedure being in the region of 205ms (Widaman, Geary, Cormier & Little, 1989).

The learning processes in arithmetic were examined by Frensch & Geary (1993). The two learning processes specified were 'strengthening' and 'composition.' Strengthening was defined as the gaining of greater efficiency in performing procedural components through learning and practice; composition was defined as achieving greater efficiency in representing sequences of procedural components in memory. They concluded that encoding and number-fact retrieval became asymptotic in adults in that accuracy and latencies improved to a stage where there was little room for improvement. Interestingly, and pertinent to the present study, it was suggested that, with regard to carrying, which did not become asymptotic, composition may not be automatic but may depend on conscious processing [involving executive functions?]. A more naïve possibility may be that carrying is less well practised than encoding and retrieval (Frensch & Geary, 1993). Nevertheless, it is perfectly plausible that, in the case of adults, retrieval of number facts has been implemented, either consciously or subconsciously for many years and has long become asymptotic. Carrying, on the other hand, is not implemented at the same level of automaticity; hence, such an operation may require more profound searching of the LTM and consequently is far less likely to become asymptotic.

Klein *et al* (2010a) studied carrying within the context of double-digit *addition* and investigated the hypothesis that there would be an effect of problem size on carrying. In other words the effect of carrying would be more pronounced in problems with large values in comparison to those with small values. Large problems take more working memory resources compared with small ones; moreover, problems demanding carrying procedures also demand more working memory resources than those that do not. It follows that large problems demanding carrying therefore demand even greater quantities of working memory resources. Klein *et al* (2010a) also tested the hypothesis that the carry effect would follow a continuum based on the sum of the units-digits of a 2 x double-digit problem (e.g., 46 + 35) as well as procedural complexity. In this study, each addition problem was presented in the centre of the screen with a response probe underneath. Participants were instructed to press a specific key to verify or otherwise whether the response probe was correct or not. The results suggested a main effect of carrying and a main effect of problems size, both in terms of increased latencies. Moreover, the carry-effect was more pronounced for larger problems suggesting that such an effect may be driven by magnitude processing. It was explained in terms of Dehaene's triple code model of numerical cognition in that on an analogue representation, decades and hundreds take more working memory resources than units. It was hence concluded that the carry-effect in complex addition may be determined by the size of the decade digit (Klein *et al*, 2010a). These findings were verified by a follow-up fMRI study (Klein *et al*, 2010b). Interestingly, this follow-up investigation showed activation in the language processing areas of the brain, particularly Broca's Area, when participants focussed on the procedural aspects of carrying, suggesting one might use inner speech to keep a track of carrying procedures. With respect to the numerical aspects of carrying, more activation was present in the area of the intraparietal sulci as the problem sizes increased; the intraparietal sulci are thought to be involved in magnitude processing (Klein *et al,* 2010b).

From a theoretical perspective, and extrapolating from the findings of Klein *et al*, (2010a, 2010b), they were suggesting that the procedural processing of carrying, if it were to be related to working memory, may be partially controlled by the phonological loop – both the phonological loop and Broca's area have a rôle in processing language. This being the case, one might expect the articulatory rehearsal system to temporarily store intermediate results and the values of each carry as procedures within a complex division problem. Kalaman & LeFevre (2007) had found that carrying caused greater demands on working memory; they used a letter-string memory task as the secondary activity, suggesting the working memory components were likely to be the visuo-spatial sketchpad (to visualise the pattern of letters) and possibly the articulatory system (to store the 4-letter strings).

The present study was designed with the intention of examining executive inhibitory functions rather than the slave systems; therefore it is inhibition that will be focussed upon. This is the first time prepotent response inhibition and resistance to distracter interference, as specific executive abilities have been studied, in relation to carrying, within a behavioural paradigm; moreover, the carrying is within the context of complex division, rather than addition, subtraction or multiplication. Within the division algorithm, a decision has to be made whether or not to carry, based on whether or not the dividend is a multiple of the divisor. If the dividend is a multiple of the divisor then no carrying needs to be employed, if it is not, a subtraction procedure may be used to determine the value of the carry. The question arises with regard to the specific rôle of inhibition within the carrying procedure: is it to monitor numerical values or procedural decisions? The results of Experiment Three indicated that inhibitory control had more impact on the procedural aspects of short-division rather than numerical attributes; the results of Experiment Four were intended to reveal whether or not this was the case with regard to the carrying procedure.

If the type of inhibition previously referred to when solving complex addition and multiplication problems (Fürst & Hitch, 2000; Imbo, Vandierendonck, & Vergauwe, 2007) is prepotent and does play a part in monitoring the carrying process, then it was predicted that both RTs and errors should increase during the prepotent response inhibition load in the results for the present experiment. The results of Experiment Three did not support the original hypothesis that RDI would result in a speed-accuracy trade-off when isolated short division procedures were carried out. However the present experiment was designed to isolate a different procedure: that of carrying. The results of the second experiment were again taken into account when formulating the prediction that RDI would cause a speed-accuracy trade-off effect in that RTs would increase from control to RDI but the error-rate would diminish. This was done as a result of the notion developed from the results of the third experiment that the responsibility of RDI was probably to monitor procedures rather than numerical values.

Klein (2010a, 2010b) suggested that, when examining addition, the size of the problems had an impact on the carrying process; carrying was implemented at a slower rate when problems were larger, owing to greater load on magnitude processing parts of the brain (Klein *et al*, 2010), there was no reference to executive functions. To investigate any possible effects on the carrying process within the context of division coupled with PRI and RDI, the problems were separated into small (divisors: 2, 3, 4 and 5) and large (divisors: 6, 7, 8, and 9) and examined in terms of latencies to rule out, or otherwise, the problem size effect having an interactive effect with PRI and RDI. One would expect there to be no PRI/RDI x problem size interaction if the carry effect (Klein, 2010a) is dependent on phonological and visual resources rather than executive control. One other, and continuing, phenomenon of interest within the context of carrying was the Friedman & Miyake (2004) notion that PRI and RDI should correlate; the results of experiment 3 suggested they did; this was within the more numerical context of short division – whether they do within the procedural context of carrying remained a question to be answered when RDI and PRI were compared at the end of the Results section.

EXPERIMENT 4

Experiment 4 was designed to test the following hypotheses: 4.1) PRI load will cause interference when the carrying procedure is implemented, consequently the RTs will increase; 4.2) error-proneness will increase when the PRI system is suppressed; 4.3) the latencies will increase rather more than PRI, when problems are subjected to the RDI condition, possibly as a result of this type of inhibition being triggered rather than suppressed; (4.4) the error rate will reduce as a result of RDI, which, in combination with the increase in RTs (Hypothesis 4.3), will result in a speed-accuracy trade-off; and, (4.5) that there would be no interactive effect between PRI and RDI, and the problem size effect. A further hypothesis (4.6) predicted that PRI and RDI would correlate within the context of implementing carrying procedures.

Method

Design

The main part of the experiment was in the form of a 3 (0 vs. 1 vs. 2 carries – arithmetic factor) x 3 (Control vs. PRI vs. RDI - cognitive factor) design. As for Experiment Three, the RTs for simultaneous secondary task (saying the direction of the arrows) were analysed separately in terms of four levels (control [direction] vs. 0 carries vs. 1 carry vs. 2 carries). Errors in the form of responding to the arrows in the PRI condition, and the control (direction) errors were analysed separately as non-parametric data. All conditions were varied entirely within-subjects; response times (in milliseconds) and error-rates were used as dependent measures. The cognitive conditions were counterbalanced in the same way as for Experiment Three.

Participants

Thirty-eight participants volunteered who were recruited from the Schools of Health and Social Sciences, Games, Computing and Creative Technologies, Arts, Media and Education, and Bolton Business School at the University of Bolton. Owing to excessive machine errors, the data from two participants was discarded, leaving thirty six in total. Fifteen participants were female and 21 were male; their approximate mean age was 28 years and ranged from 18 to 55; all were fluent in English and had normal or corrected to normal vision. None had participated in experiment three and none withdrew following the initial screening and practice process.

Stimuli

The stimuli were exactly the same as those used in experiment 3 except for those specified under the no-carry condition; these were discarded as superfluous: participants would only have entered a series of zeroes, resulting in a meaningless batch of latencies, and probably with very few errors. Hence, there were 48 division problems in total, 16 different problems for each of three groups: group 1, for the simultaneous condition; group 2 for the prepotent response inhibition condition and for the resistance to distracter interference conditions; and group 3, for the control condition (see Appendix III). Taking into account that each problem was split into three sub-problems, 48 responses were required in each cognitive condition, i.e., 192 single-digit responses for the main experiment. Within each group, 8 problems (16 sub-problems) were present forming two arithmetic conditions: one carry and two carries. Under the one-carry condition, a problem, for example, 1322/2 was split into 13/2, 12/2 and 2/2 whereas, under the two-carry condition, a problem such as 1572 was split into 15/6, 37/6 and 12/6. The same presentation format as for experiment 3 was used for each of the cognitive conditions except that, because the responses were to be remainders (r) rather than integer-answers, the integer answers were provided. The integer answers were provided in order to focus participants' attention solely onto calculating the value to be carried and nothing else; cognitive resources were therefore targeted more specifically (see Figure 6.1).

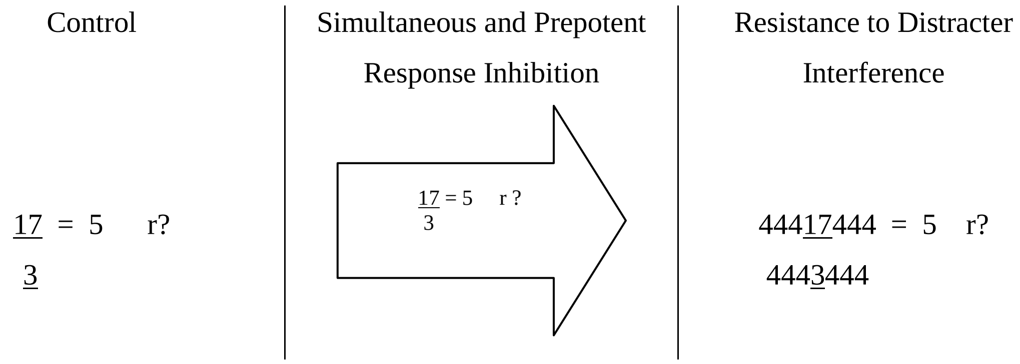


Figure 6.1. Presentation of the Problems in each Cognitive Condition

Apparatus

The stimuli were presented on the same equipment as for experiment 3.

Procedure

The procedure was the same as for experiment 3 except that the 'no-carry' arithmetic condition was discarded. Also, participants were instructed to type-in the single digit remainder rather than the integer answer to the problem.

Results

Response Times

As for Experiment Three, the Simultaneous and control (direction) conditions were analysed separately from the rest of the experiment as these were designed as a manipulative activity to induce a prepotent response, and as a comparative condition with the arrow-response in the simultaneous condition, respectively; these can be found in Appendix V and will not form part of this results section.

Only the latencies of the correctly solved problems coupled with correct responses to the direction of the arrows (in the case of the simultaneous condition) were analysed. Consistent with experiment 3, for the initial part of the analysis, the simultaneous condition responses were eliminated. Prior to analysis, the data were examined and machine errors, owing to coughs and the failure of the microphone to record responses, were removed from the analysis resulting in 8.5% of the data being discarded. The remaining data were screened participant-by-participant; any data ±2 standard deviations away from the mean were replaced by the mean; this accounted for 4.34% of the data.

Table 6.1.

Descriptive Statistics (remainders): Control, PRI and RDI (N = 36)

Cognitive Factor	Arithmetic Factor	Mean RT (ms)	SD
Control	*One carry*	2107	702
	Two Carries	2337	831
PRI	*One carry*	1967	657
	Two Carries	2369	820
RDI	*One carry*	2053	759
	Two Carries	2678	1159

PRI: Prepotent Response Inhibition. RDI: Resistance to Distracter Interference.

A 3 x 2 repeated measures ANOVA revealed a significant main effect of cognitive factor (control vs. PRI vs. RDI), $F(2, 70) = 3.95$, $p = 0.024$, $\eta^2_p = 0.10$ and a significant main effect of arithmetic factor (one carry vs. two carries), $F(1, 35) = 42.42$, $p < 0.001$, $\eta^2_p = 0.55$. There was also a significant cognitive factor x arithmetic factor interaction, $F(2, 70) = 7.05$, $p = 0.002$, $\eta^2_p = 0.17$, reflecting the comparatively selective main effect of PRI and RDI and the more pronounced main effect of the number of carries. A series of four *post hoc* tests also indicated that the interaction was as a result of the significant increase in RTs from the control condition to the RDI condition when two carries were required, $t(35) = -3.97$, $p < 0.001$, whereas, antithetically, none of the other *post-hoc* comparisons reached significance (i.e., control vs. PRI for one-carry, control vs. PRI for two-carries; and control vs. RDI for one-carry). A Bonferroni correction, which was *not* implemented, would have made no difference to these results. All tests were one-tailed: either significant increases or significant decreases, only, were of interest. Hypothesis 4.1, that RTs would increase as a result of PRI was refuted but hypothesis 4.3 which predicted latencies would increase as a result of RDI was partially supported, i.e., when two carries were required. These analyses (see figure 6.2) suggest that the difference in latencies may be designated to resistance to distracter interference rather than prepotent response inhibition.

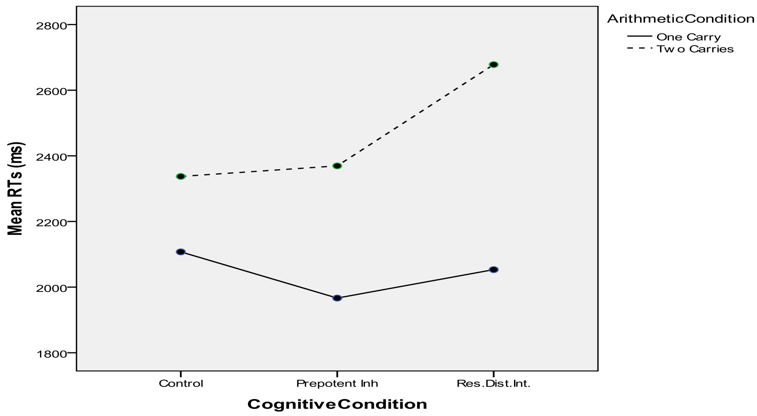


Figure 6.2. Mean Latencies

Errors

A 3(control vs. PRI vs. RDI) x 2(1 carry vs. 2 carries) repeated measures ANOVA on error-rates revealed a significant main effect of cognitive factor $F(2, 70) = 4.90$, $p = 0.01$, $\eta^2_p = 0.12$ and a significant main effect of arithmetic factor, $F(1, 35) = 10.28$, $p = 0.003$, $\eta^2_p = 0.23$. No significant cognitive factor x arithmetic factor interaction was apparent, $F(2, 70) = 0.018$, $p = 0.98$. Figure 6.3 shows the relative pattern of the errors: control vs. RDI had a far greater effect than control vs. PRI and, as expected, the two-carry problems were clearly more error-prone than those demanding only one carry. The results did not support hypothesis 4.2 which predicted an increase in error-rates as the effect of PRI but it did support Hypothesis 4.4 stating that RDI would have the effect of reducing error-rates. The lack of an interaction reflects the parallel effects of PRI and RDI on error-rates regardless of the number of carry operations.

Table 6.2

Mean Percentage Error-rates (remainders): Control, PRI and RDI (N = 36)

Cognitive Factor	Arithmetic Factor	Error-Rate (%)	SD
Control	*One carry*	3.24	4.35
	Two Carries	5.09	4.89
PRI	*One carry*	3.24	4.12
	Two Carries	4.98	5.35
RDI	*One carry*	1.62	3.76
	Two Carries	3.59	5.19

PRI: Prepotent Response Inhibition. RDI: Resistance to Distracter Interference.

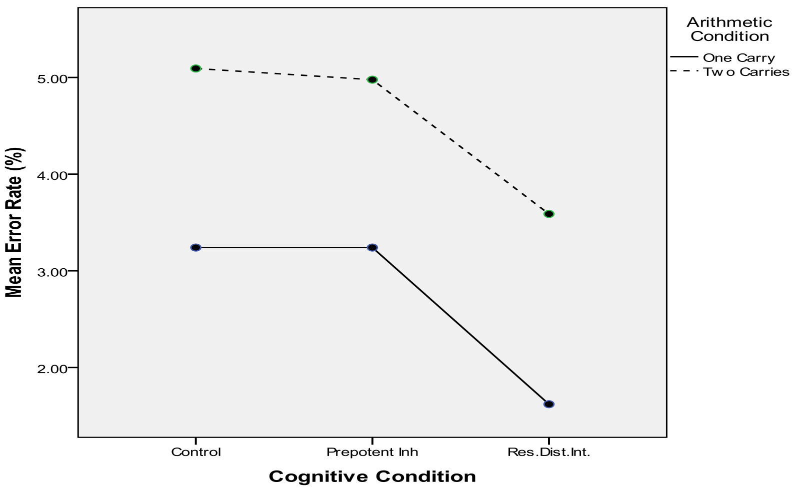


Figure 6.3. Mean Percentage Error-Rates

In order to confirm the above results, decomposition of the main effect of RDI was carried out. A 2 (control vs. RDI) x 2(1 carry vs. 2 carries) ANOVA was undertaken which revealed a significant main effect of RDI, F(1, 35) = 7.81, p = 0.008, η^2_p = 0.18 and a significant main effect of arithmetic condition, F(1, 35) = 9.39, p = 0.004, η^2_p = 0.18. No significant interaction was evident, F(1, 35) = 0.008, p = 0.93. A pair of corrected *post hoc* tests indicated that Control (M = 3.24%) vs. RDI /1 carry (M = 1.62%) represented a significant reduction in error-rates as an effect of RDI *t*(35) = 1.72, *p* = 0.0475, α = 0.05, however, the comparison on control (M = 5.09%) vs. RDI/2 carries (M = 3.59%) only approached significance, *t*(35) = 1.97, *p* = 0.0285, α = 0.025. This confirmed further support for hypothesis 4.4 that there would be a reduction in error-rates as a result of RDI.

The marginally significant reduction in errors (two carries) as an effect of RDI, coupled with the significant increase in latencies (two carries), also as an effect of RDI, both have a strong trend towards supporting the hypothesis (4.4) that there would be a speed-accuracy trade-off, as a result of RDI – but for two-carries only.

Extra Analysis

To investigate any possible number size effects on the carrying process within the context of division, coupled with PRI and RDI, the problems were separated into small (divisors: 2, 3, 4 and 5) and large (divisors: 6, 7, 8, and 9) problems and the RT data was analysed. The means and standard deviations are displayed in Table 6.4. A 3(control vs. PRI vs. RDI) x 2(small vs. large problems) x 2(1 vs. 2 carries) ANOVA on RTs revealed no significant main effect of problem size F (1, 35) = 1.96, p = 0.17. There was a main effect of PRI and RDI, $F(2, 70) = 3.86$, $p = 0.026$, $\eta^2_p = 0.10$ and of the number of carries, $F(1, 35) = 41.81$, $p < 0.001$, $\eta^2_p = 0.54$ as in previous analyses. There was no significant main effect of problem size, $F(1,35) = 1.96$, $p = 0.17$, reflecting the faster RTs for *one-carry* problems with large divisors and slower RTs for *two-carry* problems with large divisors across all levels in the cognitive factor. There was a significant interaction between problem-size and the number of carries, F (1, 35) = 81.68, $p < 0.001$, $\eta^2_p = 0.70$, indicating problem size had an opposite carry effect depending on whether one or two carries took place. Also, there was a significant interaction between arithmetic and cognitive factors as apparent in the first analysis in this results section. There was no cognitive factor x problem size interaction, $F < 1$, which was in-line with prediction (hypothesis 4.5), neither was there a three-way interaction, $F < 1$.

Table 6.3

Descriptive Statistics for Mean RTs (Cognitive, Arithmetic & Problem Size Factors)

Cognitive Factor	Arithmetic Factor	*Problem Size*	Mean RT (ms)	SD
Control	One Carry	*Small*	2232	791
		Large	1983	649
	Two Carries	*Small*	2232	798
		Large	2454	930
Prepotent	One Carry	*Small*	2143	807
Response		*Large*	1792	545
Inhibition	Two Carries	*Small*	2237	815
		Large	2502	852
Resistance to	One Carry	*Small*	2237	909
Distracter		*Large*	1875	667
Interference	Two Carries	*Small*	2580	1163
		Large	2774	1111

Comparison Between PRI and RDI in Terms of Carrying Operations

Using a similar *modus operandi* as that of Experiment Three, PRI and RDI were compared, as separate analyses to investigate any significant differences in the effects of RTs and error-rates. It has already been highlighted in the results of Experiment Three that RTs were significantly longer when RDI was implemented than under PRI load, when problems required two carries. The error-rate, however, was increased under RDI for two carries and reduced for no carries, in comparison with PRI. When the comparisons across Experiments One and Two took place in Chapter Four, it was notable that RDI slowed RTs more than PRI and, unlike PRI, RDI reduced the error rate. The first two experiments examined division as one long multi-procedural process. Experiment Three extracted the short-division procedure – was the short division procedure *partially* responsible for the lengthened RTs under RDI with regard to the whole division process? It looks possible that it was. The same could not be stated with respect to error-rates: there was an increase rather than a reduction for problems involving two carries. To ascertain whether the carry-operation may have been at least partially responsible for the behaviour of the RT and error data in experiments One and Two, similar comparisons were made here, in order to test the hypotheses (4.3) that RDI would slow RTs more than PRI and

(4.5) that PRI and RDI will correlate, both in terms of latencies and error rates. Latencies were compared first.

Multiple corrected one tailed paired sample t-tests on the RT data revealed one significant difference: an increase in mean RTs, from prepotent response inhibition to resistance to distracter interference, when two carries where required, $t(35) = -2.61$, $p = 0.0065$, $\alpha = 0.025$; a similar result was reported in experiment three. This partially supported hypothesis 4.3 that RDI would lengthen RTs more than PRI load.

The latencies were subjected to a Pearson's two-tailed correlation analysis. This indicated significant correlations between data in the same arithmetic conditions: PRI vs. RDI (one carry), $r = 0.71$, $p < 0.01$; PRI vs. RDI (two carries), $r = 0.78$, $p < 0.01$; these RT results went towards supporting hypothesis 4.6, which predicted that PRI and RDI worked together and would therefore correlate.

Errors

The errors rates were compared using similar analyses. This time, a series of independent samples t-tests on the data revealed no significant differences, $p > \alpha$. The correlational analysis indicated a significant correlation between PRI and RDI when two-carries were required, $r = 0.59$, $p < 0.01$; no correlation was evident, in relation to one carry, $r = 0.004$. The results partially supported hypothesis 4.6.

The overall prediction that PRI and RDI would correlate, in terms of latencies and errors, was mostly supported. The lack of a correlation with regard to the error data (one carry) may reflect the rather low participant error rate with regard to the problems or an example of where the problem difficulty is not intensive enough to benefit from the assistance of both PRI and RDI.

Discussion

The aim of Experiment Four was to study specifically the effect of prepotent response inhibition and resistance to distracter interference on the carrying procedure within the complex division process. The effects of the direction-stating activity were considered less important for the purpose of this experiment for the reasons stated in the discussion for Experiment Three. The results for this manipulation activity do again suggest that it served its purpose and are provided in Appendix III. The discussion will therefore focus on the induced inhibitory conditions.

The first two hypotheses (4.1 and 4.2) predicted that there would be a slowing of processing (resulting in slowed RTs) and an increased error-rate as effects of PRI load. These hypotheses were not supported, suggesting little or no rôle for PRI in terms of monitoring carrying procedures. However, it ought to be emphasised that error-rates were very low and it may initially be argued that this is a reflection of the lack of difficulty inherent in the task participants were asked to carry out for this particular experiment. The inherent lack of difficulty leads to another possible reason for there being no effect of PRI. The lack of difficulty may have been an inherent side-effect of attempting to separate the carrying procedure from the complete division process. The slowing of RTs under PRI load has been discussed from the perspective of the results of Experiment One. This slowing happened for one and two carries only, suggesting consistency with the proposals by Fürst & Hitch (2000) and Imbo, Vandierendonck, & Vergauwe (2007) that the central executive was involved in inhibiting a no-carry procedure when a carry procedure needs to be undertaken. The results of the Experiment Four are not consistent with this and suggest that for PRI to be activated in the monitoring of the carrying procedures, they must be part of a complete arithmetic process as undertaken in the first two experiments in the present study.

Another possible explanation for this inconsistency of results between Experiments One and Four is the proposal by Frensch & Geary (1993) that carrying depends on conscious processing. And, moreover, that conscious processing depends on executive functions. Participants were informed that problems either required one or required two carries for the present experiment, they also had been informed, at the outset, that the sub-problems they were asked to solve were a break-down of complete problems that involved either one or two carrying operations. They therefore did not have to make a conscious decision whether or not to carry; this decision, however, had to be made by participants in both Experiments One and Two, where problems were ordered randomly rather than being blocked by type, as in the present experiment. This being the case, this would provide further support for the notion that, for PRI to be activated then the carrying procedures should be part of a complete problem that has not been broken down into separate procedures.

Hypothesis 4.3 suggested there would be an increase in the interference effect as a result of RDI and was partially supported. With regard to latencies under RDI and assuming that the presence of flanker digits actively triggers this type of inhibition, it was evident from the *post hoc* tests that this caused slowing of processing only when two carries were required. It has already been proposed in the discussion for Experiment Two that RDI is a proactive inhibitory mechanism that is sensitive to problem difficulty. This being the case, one might argue that two carry problems take more processing resources than one carry problems (and this is evidenced by the consistently elevated RTs and error-rates when comparing two-carry with one-carry problems in all the cognitive conditions), hence it follows that RDI will reduce its intensity for less demanding problems.

Another explanation may be with reference to the required responses, in this case the values to be carried. Imbo, Vandierendonck, & Vergauwe (2007) proposed that previous responses may interfere with the response to the problem immediately being solved. In order to examine the feasibility of this explanation within the context of the present experiment, it is necessary to examine more closely the types of responses required under the one-carry and two-carry arithmetic conditions. One-carry problems contained three sub-problems; two sub-problems demanded zero responses, one sub-problem a response of 1 or a whole number greater than 1; either the first or the second sub-problem demanded a zero response and either the first or second problem expected a response of 1 or more; the third sub-problem always demanded a zero response. Two-carry problems, although procedurally more demanding, comprised two sub-problems (always the first two) demanding a non-zero response and a third sub-problem expecting a zero response; this pattern was the same and rhythmical for all two-carry problems. The results partially supported hypothesis 4.3 that RDI would slow RTs significantly; this was partial in that it only held for two carries. There is a suggestion, here, that where there is a regular rhythm of two non-zero responses followed by one zero response, then RDI may be activated to filter possible non-zero responses for every third sub-problem. However the error-rate analyses suggest a slightly different picture and was consistent with prediction (hypothesis 4.4): there were significantly fewer errors, compared with similar problems in the control condition, when one carry was required suggesting the possibility that RDI was activated to filter unwanted responses owing to the slightly randomised nature of the order of the first two sub-problems: zero responses might be more likely to interfere with non-zero responses under such circumstances. There was a *marginally* significant reduction in error-rates as a result of RDI for two-carry problems; this, coupled with no significant increase in latencies might suggest a less intensive activation was necessary because the response types (non-zero → non-zero → zero) were rhythmically more predictable, nevertheless, some activation was beneficial. This reasoning provides some feasibility with regard to the notion of filtering interference from previous responses (Imbo, Vandierendonck, & Vergauwe, 2007).

This reasoning also adds weight to the suggestion that resistance to distracter interference has a filtration effect that increases as problem-difficulty rises. The more pronounced increase in RTs, compared with PRI adds further support to the suggestion that there is a proactive control mechanism that monitors interference at the outset (Botvinick *et al*, 2001, Braver *et al*, 2007 &

Braver *et al*, 2009). RDI may be at least a part of this proactive mechanism. The increase in filtration effect as problem difficulty rises also suggests that this proactive mechanism is one that is variable with the ability to intensify or otherwise, as and when necessary. What type of unwanted intrusions are being filtered remains, for the moment, debatable, but are likely to be a combination of procedural decisions, e.g., to carry or not (Fürst & Hitch, 2000; Imbo, Vandierendonck, & Vergauwe, 2007) and previous responses that may interfere with the response to the sub-problem being immediately solved (Imbo, Vandierendonck, & Vergauwe, 2007).

Similarly to Experiment Three, the correlational analyses suggested that PRI and RDI are related when applied to separate procedures within the whole division process. They are also related in terms of providing interference as indicated by the correlating RT data, pointing to a rôle for both types of inhibition in monitoring the carrying procedures. The t-tests provided further partial support for hypothesis 4.3 in that RDI slowed the two-carry processes more than PRI load. The lack of a significant correlation between the error data for one-carry suggests a differing responsibility for each type of inhibition in carrying, possibly because RDI has more of a rôle in monitoring numerical values, whereas, looking back to Experiment One, PRI has more of a procedural capacity in monitoring carrying decisions. Regarding two carries, however, the significant correlations for both RTs and error-rates might, speculatively, be a case of PRI and RDI working together to filter the tendency not to carry.

The analysis including problem size did not seem to have a great deal of relevance in terms of the effects of the two types of inhibition, the interactions were between the arithmetic conditions and the problem sizes. The effects here were purely as a result of processing the size of the values rather than any attribution to PRI or RDI; even the number of carries did not have any interactive effect with the problem size. These results were consistent with the notion that these carry effects owing to number size were determined by magnitude processing rather than by any working memory load (Klein *et al*, 2010a, 2010b).

In summary, the analyses for the present experiment, coupled with some observation in hindsight from Experiment One, suggest a shared responsibility for both types of inhibition in terms of monitoring procedures. RDI appears to be at least part of a proactive inhibitory mechanism which is proactive in that it has been triggered by the flanker digits. The evidence points to this inhibitory mechanism being variable and having the ability to intensify as problem difficulty rises. PRI, on the other hand, may be looked upon as part of a *reactive* inhibitory mechanism which intervenes in order to monitor cognitive procedure as carrying needs to be implemented or not (*cf.*, Botvinick *et al*, 2001, Braver *et al*, 2007 & Braver *et al*, 2009). With regard to carrying, the results of the present experiment suggest that PRI has little if any responsibility to monitor numerical values, in contrast to RDI, however other evidence from earlier in the present study is not necessarily consistent with this view.

A point that will be left for the general discussion is that within the arithmetic division procedure, prepotent response inhibition is a function that is enacted by the cognitive system. However, there is no evidence provided within this chapter or previous chapters that resistance to distracter interference is intentionally employed in human arithmetic processing, only that if it is used, one may infer, particularly from the results from Experiments Two and Four that it is beneficial in terms of accuracy. It appears to be during the carrying procedure that RDI causes something close to a speed-accuracy trade-off, similar to those first evident in Experiment Two but whether carrying on its own could produce the effects evident in Experiment Two, is debatable. Other points for the general discussion are not only the similarities in the effects of PRI and RDI from one experiment to another, but also the inconsistencies, some of which are as a result of the differences in the way the cognitive processing system must react to extended multi-procedural processes as opposed to relatively straightforward shorter single digit calculation.

CHAPTER SEVEN

GENERAL DISCUSSION

Over the course of the present study, four experiments are reported that were designed to answer two broad questions: Are prepotent response inhibition and resistance to distracter interference, as types of inhibition, utilised in the cognitive processing of complex division problems requiring and not requiring remainder-carrying procedures? Evidence from previous studies suggest memory updating is involved in updating intermediate results in mental arithmetic and response selection has a rôle in selecting the required response from several activated responses to problems such as 3 x 8 [21, 24 or 27?] (Deschuyteneer & Vandierendonck, 2005a, 2005b; Deschuyteneer *et al*, 2006). The central executive is (without being directly specific as to subcomponents) involved in retrieval of number-facts from LTM (Duverne *et al*, 2008; Imbo & Vandierendonck, 2007), verification of the correct response (De Rammelaere e*t al*, 2001), strategy selection (Duverne, *et al*, 2008) and for keeping track of intermediate results (Imbo & Vandierendonck, 2007). None of these refer directly to inhibition, as a component of the central executive of working memory (Baddeley, 1996) and none refer directly to PRI or RDI as subcomponents of stimulus inhibition. The present study went, at least part of the way to address this issue. The second question was: Are there any specific procedures, within the complex division process that benefit from the utilisation of these particular types of inhibition? The present study has utilised the same complex division problems and extracted the short-division and carrying procedures to try to answer this question.

7.1 Summary of Findings from the Four Experiments

Experiment One focussed on prepotent response inhibition (PRI) and employed a new type of methodology in order attempt to load the PRI system where a dual-task activity was used as a manipulation task designed to induce a prepotent response-state. All the problems comprised four-digit dividends and single digit divisors and demanded three-digit responses. When no carrying had to be implemented, as predicted, no significant difference was evident, suggesting support for the notion that, under the no-carry condition, the responses are taken directly from LTM (Fürst & Hitch, 2000; Imbo, Vandierendonck & De Rammelaere, 2007; Imbo**,** Vandierendonck & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2002). The evidence from this experiment, in terms of elevated RTs suggested that PRI has a supervisory rôle in monitoring carry operations. Evidence generated from the error data indicated that PRI also had a responsibility to filter errors if the problem difficulty was demanding enough – in this case requiring two carry operations. All participants were encouraged to solve the problems in three stages, e.g., 1734 ÷ 3 was solved by decomposing the problem into 17 ÷ 3, 23 ÷ 3 and 24 ÷ 3. The possibility was raised that PRI may inhibit exact partial-responses when part of the problem was, for example, 17 ÷ 3; the expected response to this was 5 (then the remainder of 2 would be carried and placed in front of the 3 to form 23); the digit 5, however is an approximate response. The exact partial-response being 5⅔ and the normal partial-response being 6 (**5** x3 = 15 whereas **6** x3 = 18; 17 is closer to 18 than it is to 15), hence either 5⅔ or 6 would need to be inhibited. Logically, it follows that PRI might be involved in inhibiting approximate partial responses when an exact response is required, but, as the two-carry condition contained twice the number of approximate partial-responses compared with exact partial-responses, the former is more likely; two consecutive approximate responses might raise expectations that the third response should be approximate as well.

Experiment Two loaded the resistance to distracter interference (RDI) system by extending the Eriksen Flanker Task (Eriksen & Eriksen, 1974), in numerical form, to make it congruous with the format of the division problems. The flanker digits caused a trend towards slower, more accurate responses possibly as a result of conflict monitoring. Conflict monitoring tends to result in speed-accuracy trade-off (cf. Botvinick *et al*, 2001) and the evidence suggested the speed-accuracy trade-off was at its most intensive when one carry was required. This was a contrast to the results in Experiment One where the most intensive effect, according to both dependent variables, was when two carries were required; there was a significant *elevation* in both latencies and error-rates for this condition, suggesting differing rôles for each type of inhibition. When the results of Experiments One and Two were compared, the considerably longer RTs under the RDI condition, in comparison to those for PRI, suggested a proactive rôle for RDI that was triggered by the flanker digits, and a reactive responsibility for PRI.

The third experiment represented an attempt to extract one of the procedures of the complex division process: short division. At first sight, the results for this experiment were all but inconclusive but the opportunity to carry out a more *forensic* analysis of the data was provided by separating the sub-problems into those with divisible and non-divisible dividends, e.g., 17 ÷ 5 is non-divisible because it leaves a remainder and provides an approximate response, whereas, 15 ÷ 5 is divisible and provides an exact response. It was proposed that the rôle of PRI was numerical in that it had a responsibility to inhibit approximate responses where exact responses were required, as evidenced by the significant elevation in latencies for sub-problems with divisible dividends under the two-carry arithmetic condition. This was consistent with the conclusion derived from the evidence produced during Experiment One. Another related interpretation was that PRI was a reactive type of inhibition that was procedural in that it monitored breakages of prepotent expectation. Both these interpretations were evidenced by the pattern of sub-problems in the two carry condition: non-divisible, non-divisible, divisible, hence, every third response would need to be monitored to filter any possible approximate responses. However, this assertion needs to be treated with caution as the question arises as to whether or not two consecutive approximate responses is enough to induce prepotent tendencies. RDI slowed responses for the no-carry and one-carry conditions prior to separating divisible and non-divisible sub-problems, suggesting, particularly under one-carry, that RDI was monitoring approximate/exact response conflict – the pattern of exact/approximate responses being less predictable than under two carries. This represented a subtle but, nevertheless, discernible difference in rôle between PRI and RDI. The correlations between the individual mean RT and error-rate patterns were very strong, supporting the proposal by Friedman & Miyake (2004) that PRI and RDI should correlate. From the evidence of overall longer RTs caused by RDI, in comparison to PRI and the significantly slower RTs for divisible sub-problems within the two-carry condition, it was proposed that RDI was a proactive mechanism that filtered approximate responses when exact ones were required and *vice-versa*. The ANOVAs on the error-data were inconclusive, reflecting the relatively less cognitively taxing solving of sub-problems rather than complete division problems in Experiments One and Two.

Experiment Four used the same methodology as for Experiment Three to investigate the behaviour of carry operations except that the no-carry arithmetic condition was dispensed with; if it were not, all responses would have been superfluous zero responses. In the case of RDI, the evidence in terms of a significant reduction in error-rates for one carry with no significant difference in latencies, coupled with a marginally significant reduction in errors for two carries with a significant increase in RTs pointed to this inhibitory mechanism being variable and having the ability to intensify as problem difficulty rises. It could also be proposed, from the results of Experiment Four, that RDI may monitor interference from previous responses or the previous response (cf., Imbo, Vandierendonck, & Vergauwe, 2007). It was only for the two-carry condition that there was a speed-accuracy trade-off; ironically this SAT phenomenon affected the *one-carry* condition most intensively in Experiment Two. PRI, on the other hand, at first sight looked like it may not have a rôle in monitoring carry operations. However, from the evidence collected in Experiment One, it may be looked upon as part of a *reactive* inhibitory mechanism which intervenes in order to monitor 'cognitive procedure' as carrying needs to be implemented or not, but only over the course of the complete division process (cf., Botvinick *et al*, 2001, Braver *et al*, 2007 & Braver *et al*, 2009); it may have been the case that the activities in Experiment Four were not sufficiently proceduralised for PRI load to cause any effects. With regard to carrying, the results of the present experiment suggest that PRI has little if any responsibility to monitor numerical values, in contrast to RDI, however other evidence from earlier in the present study is not necessarily consistent with this view. The correlations between the individual mean RT patterns were very strong. However for the mean error-rates, there was only a strong correlation for two carries. This was mainly consistent with the proposal by Friedman & Miyake (2004) that PRI and RDI should correlate. The overall error rate was very low and either the absence of a correlation for one carry may have reflected this or it might have been because RDI prevented errors under the one carry condition as the sole inhibitory mechanism without the assistance of PRI. The latter interpretation is feasible as PRI load made no significant difference in terms of RTs or error-rates when one carry was required.

In summary, examining inter-experiment consistencies first, RDI elevated the latencies throughout the four experiments significantly more than was evident as an effect of PRI. This provides weight to the proposal that RDI is a proactive type of inhibition whereas PRI is reactive. The SATs and extended latencies as an effect of RDI suggest that this is the inhibition-type that does continuously monitor the division process, adding further strength to the proposal that RDI is proactive. The evidence from Experiments One and Three support the notion that PRI inhibits approximate responses when exact responses are required, particularly where there was a regular pattern of response types; PRI had the most intensive effect on the problems requiring two carries. However, owing to the short series of approximate responses, concern was raised over whether two consecutive approximate responses were enough to induce this as a prepotent response-type. Future research might use a similar technique to examine division problems with five and six-digit dividends and examine patterns such as four approximate responses followed by one exact response to determine whether or not similar effect could be observed. RDI had the most significant effect when problems required one carry; under one carry the responses, the pattern of exact and approximate responses was less predictable, suggesting an increased chance of interference from different response-types, as opposed to a prepotent response pattern.

Turning to inconsistencies, one such was between Experiments Two and Four in that the speed-accuracy trade-off in the second experiment was at its most intense when one carry was required but the only similar result in Experiment Four related to the two-carry condition. This suggests that the SATs in Experiment Two cannot be attributed purely to interference control on carrying only; other procedures such as the subtraction procedure to find the remainder to be carried or the maintenance of intermediate results, amongst other procedures may have also played a part in the cause of the SATs in Experiment Two. Another inconsistency was the lack of evidence in Experiment Three of anything approaching a SAT, hence ruling out individual short-division processes as being a causal factor. However, this does not rule out the possibility that a series of three short-division procedures might have been slowed and compensated for with greater accuracy when the complete division process took place. Reasons for this might have been interference, in the form of competitor results when keeping track of intermediate results. Such a notion might be considered for further research, possibly using computer simulations of the complete division process.

Another point of interest when comparing PRI with RDI is the order of effects. If PRI disturbed both latencies and error-rates, it would follow that the PRI system within WM was overloaded. Furthermore, when the interaction with regard to RTs in the first experiment was reduced, *post-hoc,* to control *versus* PRI, there was a significant increase in latencies for one and two carries, only. For error-rates, the interaction was rooted in the significant increase under the PRI load for just two carries only. When combined, this adds support for reactivity, with respect to PRI: reactivity that increases with problem difficulty. On the other hand, RDI, owing to the design of Experiment Two has been triggered by the flanker digits to create a parallel increase (with the number of carries) in latencies coupled with a corresponding more or less parallel reduction in error-rates, when compared with the control condition. It must be concluded that, owing to the design of the experiment(s), RDI begins the inhibition process.

Finally, concerning the possible working together of the two sub-components of stimulus inhibition, it is evident from the results of some of the analyses in the present study that, over the long-term (i.e., several minutes), they appeared to have separate unrelated rôles. For shorter procedures, there is strong evidence that, as predicted, both RDI and PRI work together and may be part of a shared mechanism for goal maintenance in a situation where there is much external distracting information, such as inappropriate responses (Friedman & Miyake, 2004) or part of a common selective inhibitory system (Verbruggen, 2005). To reiterate, RDI is more proactive in nature whereas PRI is more reactive and assists RDI with conflict resolution, as deemed necessary (see Figure 7.1 for an initial diagrammatic representation).

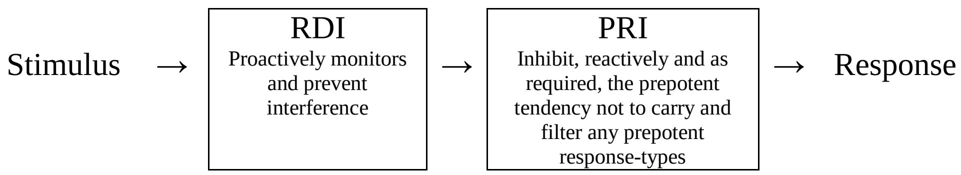


Figure 7.1. Order and Rôles of RDI and PRI (Experiments, Two, Three and Four)

7.2 Some Theoretical Frameworks with Possible Applications to the Above Findings

There are a number of possible theoretical explanations for the phenomena that were revealed by the results of the four experiments in the present study. Experiment Two produced somewhat unexpected results: the general trend was of a reduction in error-rates under the RDI manipulation, regardless of the number of carries, coupled with the predicted increase in latencies, again, regardless of arithmetic condition. This 'speed-accuracy trade-off' could be examined in terms of 'Conflict Monitoring Theory' (Botvinick *et al*, 2001) where elevated conflict, in this case caused by the flanker digits, resulted in a reduction in response priming, leading to slower but more accurate responses. As a reminder, Campbell (1997, 1999) used error-priming, i.e., priming a target trial with pre-trials from the same times-table hence, elevating the probability of participants responding with a close yet incorrect response to the target trial. No deliberate priming of responses was carried out for Experiment Two: the problems were solved in a randomised order. Any response conflict, as suggested earlier, would have been as a result of the unpredictable pattern of approximate and exact intermediate responses as evidenced in Experiment Three. This type of response conflict was present in Experiment Two whenever a problem requiring one carry was solved. It would also have been present as a result of the randomised order of no-carry and one and two-carry problems. Either way, conflict monitoring theory is a plausible explanation. Conflict monitoring theory was later extended upon by Verguts & Notebaert (2009); this was termed 'Adaption by Binding Theory.' They used a variant of the Stroop Task (Stroop, 1935) in their experiments and suggested that if, for example, the colour is the more important dimension (i.e., the colour of the word is emphasised in the instructions rather than the verbal dimension), the colour dimension is enhanced owing to top-down pressure from a 'colour-demand' neuron. The strength of the actual word / colour conflict in the response is detected by a performance monitor. These conflict signals are then sent to an area in the brainstem concerned with conflict which, in turn, sends signals throughout the cortex that learning needs to increase [to cope with the conflict] (Verguts & Notebaert, 2009).

If such a theory is applied to the adapted Eriksen flanker task in the present study, the notion is developed that each problem, hidden amongst the noise-digits, and being the more important dimension, is enhanced, in terms of focus, by a cognitive control system. Signals would then be sent throughout the cortex that learning to ignore the flanker-digits and to direct one's attention onto the problem is a priority; consequently, this resulted in slower but more accurate solutions to the problems. This theory also seems plausible and the evidence from the present study in terms of elevated latencies and reduced errors suggests RDI could be *part* of such a cognitive control system. This is a further possibility, to be explored later.

The 'Response Competition Paradigm,' as discussed in Chapter 4 (Notebaert & Verguts, 2006), provides evidence that, when undertaking a number recognition task with an increasing numerical distance between the target digit and the flankers, the result is proportionally quicker RTs. This notion cannot be specifically applied in the present study: the distances between the flanker digits and problem digits were not varied but kept as normal type-script distance to create the greatest interference; furthermore, the problems contained more than one digit. In the original Eriksen flanker task (Eriksen & Eriksen, 1974), where the flanker letters were further away from the target letter than one normally sees in conventional type-script there was also a lack of interference. The effect of flanker letters on a single target letter (Eriksen & Eriksen, 1974) and that of flanker digits on a target single digit (Notebaert & Verguts, 2006) are not dissimilar. The proposal could be made, however, that the original Eriksen and Eriksen (1974) theory could be applied to the present study. Eriksen & Eriksen (1974) maintained that when the flanker letters were present (in the closest position) participants could not prevent processing of the flanker letters; they inhibited their responses until they were able to discriminate which was the target letter. The results of the present study takes this a step further and suggest that, when flanker digits were present, participants inhibited not only the final response but also the encoding of each problem and the intermediate calculations, in a series of discriminations, until the required response was satisfactorily formulated; this could be looked upon as RDI forming part of a proactive, constantly monitoring, control mechanism. This may be inferred from the consistently elevated RTs throughout the present study, which, moreover, were significantly elevated, in comparison to latencies in the presence of arrows. Assuming executive functions to be analogous to an attentional control mechanism, part of controlled attention is what Engle and colleagues (Engle, Tuholski, Laughlin & Conway, 1999) termed goal maintenance, in the case of the present study, remaining focussed on the division problem. In order for goal maintenance to prevail, there has to be conflict resolution – conflict, in the present study being the flanker digits. As already specified in the method section for Experiment 1, participants were encouraged to execute a series of three short-division procedures as part solutions to each final response.

Assuming that responses to short-division problems may be retrieved from a network of division facts, rather like multiplication, it would be beneficial to explore some of the literature on multiplication. Campbell (1987) suggests that encoding a multiplication problem activates multiple candidate answers and that the accessibility of correct responses depends upon the simultaneous activation levels of associated false answers. Verguts & Fias (2005) took a slightly different but related approach. They suggested a model for multiplication processing where a problem such as 6 x 4 is processed via a network of facts and a decades-field is activated to produce 20 whilst a units-field produces 4. These are then recomposed to activate 24. On its journey through the network, 6 x 4 also activates 5 x 4 and 7 x 4, these being categorised as consistent neighbours: they both activate 2 in the decades-field. Six-times-four also activates 6 x 3 and 6 x 5: these activate different values in the decades-field (1 and 3) and are categorised as inconsistent neighbours, hence, 6 x 4 has two inconsistent neighbours and two consistent neighbours. Bearing in mind that multiplication is the inverse operation of division, there is a strong possibility that division may be mediated by multiplication.

If division is mediated by multiplication (Campbell, 1997, 1999; Campbell & Albert, 2010, LeFevre & Morris, 1999), it might follow that a sub-problem such as 36 ÷ 4 may be approached by reference to 4 x ? = 36. Assuming 4 x 9 not only activates 36 but also 32 (i.e., 4 x 8) one might argue that it follows that 4 x ? = 36 will not only activate 9 but also 8 – the latter response, being in need of inhibition. Further evidence that neighbouring nodes (responses) are activated is apparent from stimulus onset asynchrony (SOA) experiments, where the time intervals (SOAs) between the stimuli were varied. Rusconi and colleagues carried out a verification task experiment with different SOAs between trials. As expected, the longest SOA resulted in the shortest RTs. For the verification trials, for example, [8 3] followed by [16] (is 16 a product of 8 x 3? Y/N) had a longer RT than [8 3] followed by [17]. Also as expected, [8 3] followed by 24 led to the shortest RT (Galfano, Rusconi & Umiltà, 2003). Similar findings were reported by Rusconi and colleagues who reversed the order of the verification tasks: [24] would be followed by [8 3], (Rusconi, Galfano, Rebonato & Umiltà, 2006). This activation of 'close-responses' may, speculatively, have been another source of conflict, in addition to the flanker digits.

Much has been discussed, so far about, conflict and conflict resolution. This begs the question: What is conflict? Szmalec and colleagues (Szmalec, Demanet, Vandierendonck & Verbruggen, 2009) forwarded a comprehensive viewpoint on this subject. When cognitive processing takes place, memory activations *develop* and *extinguish* simultaneously; these simultaneous actions are closely monitored. In order to prevent behavioural errors, conflict between developing and extinguishing memory activations has to be resolved (Szmalec *et al*, 2009). This viewpoint could be applied to complex division. If complex division is regarded as a series of short division calculations, with remainder-carries to the next column on the right, as and when necessary, a number of memory activations are likely to take place. If we take the procedural activations, decisions have to be made regarding whether or not to carry (activations: carry, don't carry) and, integrating the notion that the prepotent activation is, 'don't carry,' then this will conflict with, 'do carry,' if a carry is required. A numerical activation could be regarded as what value to carry, in which case zero-remainders may conflict with non-zero remainders. Where one carry was required, in the present study, the order of these was less predictable and this resulted in longer RTs under RDI load: an example of RDI helping to resolve conflict. Any numerical activations might include candidate answers to the short divisions (e.g., 15 ÷ 3 = 5), this may also activate 4 and 6 (Campbell, 1987), both of which need to be inhibited to prevent a behavioural error.

It has been further argued that as well as being resolved, conflict needs to be detected and prevented. For detection, prevention and resolution to be implemented, some form of control mechanism has to be present. Braver *et al* (2007) developed the Dual Mechanism of Control (DMC) theory which was further supported with neuro-imaging evidence (Braver *et al*, 2009). Here, it is proposed that there exists a *proactive* mechanism to prevent interference and a *reactive* mechanism to detect and suppress interference (Braver *et al*, 2007). Bearing the DMC in mind, what is particularly striking, with regard to the first two experiments in the present study, is the difference in RTs between the RDI and PRI conditions. Those RTs for resistance to distracter interference are substantially longer than those for prepotent response inhibition. From the perspective of the present study, RDI worked proactively whereas PRI worked reactively. The latencies under RDI load were substantially longer than those under PRI load, suggesting RDI was active throughout the calculation process as could be inferred from Experiment Two. RDI was also sensitive to problem-difficulty, for example, it became more intense where one carry was required, owing to the relative unpredictability of the values to be carried. Although, RDI was less intense when two-carries were required in Experiment Two, it was PRI that was activated reactively, to filter non-zero carries when zero carries were required, as evidenced in Experiment One. PRI was more selective, for example, it became active when two carries were required, particularly where sub-problems were divisible under the two-carry condition, where the pattern of zero and non-zero carries were predictable, as the results suggested in Experiment Three. It does need to be emphasised, however, that Braver *et al* (2007) are adamant that the two control mechanisms are not inhibitory, as such. They achieve inhibition owing to active goal maintenance exerting top-down pressure on local competition within the posterior brain system.

7.2 Synthesis of Findings and Theoretical Frameworks

Under contrasting conditions and the differing nature of the tasks, depending on what such tasks were designed to ascertain, there was a variation in the intensity and presence of the two types of inhibition highlighted in the present study. Various theories have been visited that might be applied to the results of the four experiments, namely, Conflict Monitoring Theory (Botvinick *et al,* 2001), Adaption by Binding Theory (Verguts & Notebaert, 2009), Controlled Attention (Engle *et al,* 1999), Lateral Inhibition (Verguts & Fias, 2005) and Dual Mechanism of Control Theory (Braver *et al,* 2007, 2009).

Conflict Monitoring Theory (Botvinick *et al*, 2001) provides for a mechanism for detecting conflict and should result in a reduction in response priming and the consequent slower but more accurate responses. Such responses were evident in Experiment Two where there was a speed-accuracy trade-off; they were also evident in Experiment Four when two carries were required; in both cases, this was as a consequence of RDI. If one restricts oneself to Experiment Four, there may be a further explanation in that, under the two-carry condition, the third of each group of three responses is an exact rather than an approximate answer that requires no carrying. However, the RT data from Experiment Three suggests that RDI was assisted by PRI, in this respect owing to the elevated latencies under both manipulations for two carries where dividends were divisible. Moreover the SAT in Experiment Two was most intense when one carry was required. This leaves two possibilities, within the context of the present study: the first two responses, being remainders to be carried, might prime the cognitive system into the expectation that there will be a non-zero value to be entered a third time; or, the one-carry condition caused more conflict because the order of divisible and non-divisible sub-problems were less predictable. This would be consistent with the interpretation forwarded by Imbo, Vandierendonck, & Vergauwe (2007) that the carry and no-carry frames of mind may interfere with each other. These expectations have to be inhibited and it was RDI that either prevented (most) entries of non-zero values or implemented the suppression – in the case of two carries, it was assisted by PRI. Another possibility, outside the context of the present study is that the 'carry or no-carry' decision on the third response was contaminated by proactive interference rather than RDI or PRI. If this were the case, then this leaves an opening for future research involving resistance to proactive interference, or resistance to information that was relevant (i.e., the previous response) but has become irrelevant to the task being solved (Friedman & Miyake, 2004) – a type of inhibition that was not examined, in the present study.

Adaption by Binding Theory (Verguts & Notebaert, 2009) where a problem is subjected to enhanced focus that eventually results in strengthened learning might also be nominated as evident in Experiment Two, Experiment Four (2 carries) and in Experiment One (2 carries). The flanker digits caused interference from the beginning of each problem or sub-problem in Experiments Two and Four. This conflict between the flanker digits and the problem itself may have triggered a cognitive system to send a message via an area in the brainstem to the rest of the cortex with a directive that learning must take place in order to focus on the problem (or sub-problem) that was hidden amongst the noise digits. Within the perspective of Adaption Binding Theory, it does not appear that any speed-accuracy trade-off would be expected and conflict monitoring theory therefore seems more feasible, within the context of the present study.

In the case of lateral inhibition (Verguts & Fias, 2005), here, a number of close responses are activated simultaneously and for the correct response to prevail, the winner has to be the correct response (Coultrip *et al,* 1992). Within the context of values to be carried, it is speculative whether responses close to the one required would necessarily be activated. Although there is evidence that with regard to times-tables, a number of candidate responses may activated (Galfano, Rusconi & Umiltà, 2003; Rusconi, Galfano, Rebonato & Umiltà, 2006) there is no evidence that a value to be carried would also activate other small values such as 0 and 2 as well as 1. This might be a subject for further research. Furthermore, owing to the difficulty in ascertaining the precise cause of the errors, particularly when only carrying was extracted, lateral inhibition theory may be the most difficult and speculative to apply within the context of PRI, RDI and complex division.

Dual mechanism of control (DMC) theory (Braver *et al,* 2007) is not perceived as an inhibitory mechanism but one that achieves inhibition via top-down pressure on the posterior brain system. Included within the theory is a proactive mechanism to prevent interference and a reactive mechanism to detect and suppress interference. Assuming DMC to be a form of umbrella control mechanism that has its purpose in monitoring conflict and then assigning inhibitory systems to carry out suppression of conflict via cortical regions such as the parietal and occipital lobes, one might speculate that certain types of (possibly matching) inhibition might become enacted. Matching, in that the proactive mechanisms of DMC may activate proactive types of inhibition, and similarly the reactive mechanism of DMC may activate reactive types of inhibition. There do appear to be some similarities between DMC and the adaption by binding theory. The reactive mechanism to detect interference in the DMC might be compared to the performance monitor to detect conflict in the adaption by binding theory. In the DMC pressure is exerted via the prefrontal cortex on local competition in the posterior region in order to achieve inhibition; in the adaption by binding theory, conflict is detected by a performance monitor in the medial prefrontal cortex. One might attempt to synthesise these two theories but there does appear to be a difference in the detection mechanisms. Whereas the DMC has two mechanisms, a proactive one for prevention of conflict and a reactive one for detection, the adaption by binding theory appears to have a single performance monitor which measures the strength of any conflict and an area in the brainstem sends signals of variable strength throughout the cortex.

It is difficult to find evidence for the DMC directly from the results of the present study owing to DMC not being an inhibitory mechanism, as such. Hence, references to it within the context of the cognitive processing of complex division problems will be from the perspective of it being entirely an umbrella control mechanism. One might speculate, however, that matching types of inhibition will become activated as a result of downward pressure from the DMC. The results of the present study strongly support the existence of a common selective inhibitory system (Verbruggen, 2005) that comprises RDI and PRI. They also strongly support the concept of a dual inhibitory mechanism that proactively prevents interference, as in the case of Experiment Two, particularly where one carry was required under RDI load and the order of divisible and non-divisible sub-problems were comparatively erratic. Moreover, this dual inhibitory mechanism has the capacity to be reactive, as in the case of Experiment Three where RTs were lengthened under PRI load when two carries were required and, very selectively, when the sub-problems were divisible. In terms of theory, there seems to be much in common, in terms of *modus operandi* with the DMC (Braver *et al,* 2007) and the notion of PRI and RDI being part of shared mechanism for goal maintenance in the presence of interference (Friedman & Miyake, 2004) or a common selective inhibitory system (Verbruggen, 2005). The results of the present study are consistent with a dual mechanism of control, albeit one that is inhibitory at the outset.

Conclusions

Throughout the present thesis two extractable inhibitory sub-components that formed experimental conditions have been extracted from the stimulus inhibition component (Baddeley, 1996) and have been studied as separate sub-components. The methodological problems of secondary tasks loading executive components other than the one being studied have been addressed, at least to a certain extent. Experiments Three and Four represented an attempt to extract two arithmetic procedures deemed worthy of further investigation: Experiment Three investigating the effect of PRI and RDI on the short-division stages and Experiment Four examining their effect on carrying the remainder-digits; for obvious reasons, the no-carry arithmetic condition was absent from the latter experiment. Moreover, the division problems employed in Experiments One and Two formed the basis of the sub-problems used as stimuli in Experiments Three and Four. This is the first time that the solving of a set of complete division problems specifically under PRI and RDI has been studied with follow-up investigations into the effect of PRI and RDI on separate procedures, using the same problems throughout. The present study represents a simulation of part of the complex division process using human participants within a behavioural paradigm and, as will be claimed, has the potential to be a starting point for further research.

The conclusions relating to methodological problems can be summed up briefly by referring to executive load. For example, one method of loading the central executive is the use of the trails task (e.g., Imbo, Vandierendonck, & Vergauwe, 2007). It was suggested by Baddeley (1996) that this activity suppressed prepotent response inhibition; this is perfectly plausible in that a sequence such as A – Monday, B – Tuesday is the prepotent response but once the days of the week are finished and the sequence becomes I – Monday, J – Tuesday, this breaks the prepotent tendency to match the first day of the week to the first letter of the alphabet. The problem with the trails task is that it is likely to suppress response selection (day of the week vs. letters of the alphabet) and the articulatory system of the phonological loop as well as PRI. The method used in the present study where a dual task is used as a manipulative activity to condition participants into a prepotent state of saying arrow-directions and then relieving them of the vocal obligation but the arrow is still present meant participants then had to inhibit an induced prepotent tendency. The articulatory and response-selection elements are therefore extinguished. This methodology provides at least a starting point for further research into PRI, possibly on other arithmetic operations. A second example concerns the loading of interference control. The one-back two-choice reaction time task where participants respond to the *previous* tone which was either high or low is used to load memory updating (e.g., Deschuyteneer *et al,* 2006). Owing to conflict between the ‘n’ response and the ‘n – 1’ response, this might also load interference control (Deschuyteneer *et al,* 2006). The present study utilised the Eriksen flanker task (Eriksen & Eriksen, 1974) to trigger interference control. The triggering may, however, be regarded as a problem because it is not a true loading: rather than overloading the RDI system, the results of the present study suggested that the flanker digits set interference control in motion. Provided this triggering is borne in mind then the use of flanker digits as in the present study provides a method for studying RDI with a greater degree of purity than the one-back two-choice RT task.

Before proposing more specific conclusions with respect to PRI and RDI as two sub-components of the central executive it would be prudent to reiterate what has already been suggested with regard to other executive components within the context of mental arithmetic. There is evidence that other central executive components besides inhibition have a significant rôle in the mental calculation process. Memory updating is thought to be involved in simple multiplication and addition (Deschuyteneer *et al*, 2006). Its rôle is likely to be for the retention of intermediate results, in the case of simple multiplication, and where participants use a counting process rather than direct retrieval. If a counting process is used then memory updating will update the counting process until the required answer is ready for the response. With respect to the present study, memory updating is most likely to be activated to maintain results of sub-problems within the division process; it would take further research, however, to be able to support such a notion. Response selection is thought to be activated in simple addition and multiplication (Deschuyteneer & Vandierendonck, 2005a, 2005b). In the case of multiplication, it has been seen that a pair of digits to be multiplied activate several candidate responses (Campbell, 1997, 1999; LeFevre & Morris, 1999; Rickard, 2005; Rickard & Bourne, Jr., 1996, 1996; Verguts & Fias, 2005) the correct one requires selection; this is a likely rôle for response selection (Deschuyteneer & Vandierendonck, 2005b). An executive component that may work in conjunction with response selection might be inhibition; inhibition, in this case, would suppress the candidate responses that were incorrect (Campbell & Clarke, 1989). There is no evidence in the present study that PRI or RDI suppress such responses; they were more involved with the procedural aspects with regard to carrying rather than to filter any inappropriate candidate responses. This does not rule out the possibility that resistance to proactive interference (RPI) may be involved in filtering unwanted responses but this was beyond the scope of the present study. Future research might find a suitable task that loads RPI and investigate its specific involvement in mental arithmetic.

The main point of the previous digression was to highlight involvement in mental arithmetic of two other executive components, response selection and memory updating. Returning to the evidence from the present study that PRI and RDI do have separate rôles and could therefore be regarded as two separate inhibitory sub-processors, i.e., PRI and RDI, it has been inferred from the results generated by Experiment One that the speed of processing complete division problems was significantly slowed by PRI load, when carrying was required. Moreover, when *two* carry-operations were required, there were significant increases in error-rates. PRI load did not affect processing efficiency when no carries were demanded and this caused a significant interaction. Overall, this points to PRI being somewhat selective and not a type of inhibition that is necessarily involved in taking partial results directly from LTM. RDI, on the other hand, was rather less selective and slowed the processing of division problems, significantly, regardless of the number of carries; this can be determined from the results of Experiment Two. Furthermore, although significant and marginally significant reductions in error-rates were evident for problems requiring one carry and no carries, respectively, the overall trend was a speed-accuracy trade-off regardless of the number of carries. This overall trend was supported by the lack of an interaction for Experiment Two, in *contrast* with the presence of an interaction for Experiment One. Further support for there being two subcomponents is provided from the results of Experiment Four, where the carrying procedure was extracted. Here, RDI, alone, had a slowing effect where two carries were required. PRI had no discernible effect on the carrying procedure. Recent EEG evidence does suggest that PRI and RDI (or response inhibition and interference suppression) are dissociable (Brydges *et al*, 2012).

Some of the evidence needs to be treated with a degree of caution owing to similarities in the behaviour of the two types of inhibition and might be suggestive towards there being a single inhibitory sub-processor. From examination of the RT data generated by Experiment Three, it was found that both PRI and RDI were responsible for elevated latencies during the two-carry (divisible) condition. Furthermore, neither PRI nor RDI had a significant effect on error-rates prior to the arithmetic conditions being decomposed into divisible and non-divisible sub-conditions. The correlations (between the RTs for PRI and RDI) in both Experiments Three and Four were all positive, suggesting that as RTs owing to PRI load were slowed, as were those caused by RDI. Similar correlational patterns were observed with respect to error-rates. Taking these similarities, coupled with the evident slower RTs for RDI, in comparison to PRI, one might ask if there is the possibility that PRI and RDI are part of a single inhibitory process that operates to a higher degree of intensity when divisible sub-problems are encountered. Kane and colleagues (Kane, Bleckley, Conway, & Engle, 2001) suggested the possibility that PRI and RDI form a unitary construct, as did Friedman & Miyake (2004).

However other evidence suggested PRI and RDI, as two separate sub-processors, nevertheless each form part of a shared *conflict detection* and *resolution* mechanism, assuming there is a cognitive mechanism to trigger RDI. The speed-accuracy trade-off analysis in Experiment Two suggested that RDI is a process that prevents (or at least was triggered and therefore prevented) external intrusions from the cognitive channel to enable more accurate calculations. The RT analyses indicate that both types of inhibition may work together to monitor carrying processes and, if need be, filter the prepotent tendency to, if at all possible, to undertake a complex division calculation with no carrying procedures or inhibit a two-carry procedure when one-carry is required, and *vice-versa*. Figure 7.2 provides a possible hypothetical model of the calculation procedures of the complex division process that emerges from the results of the present study.

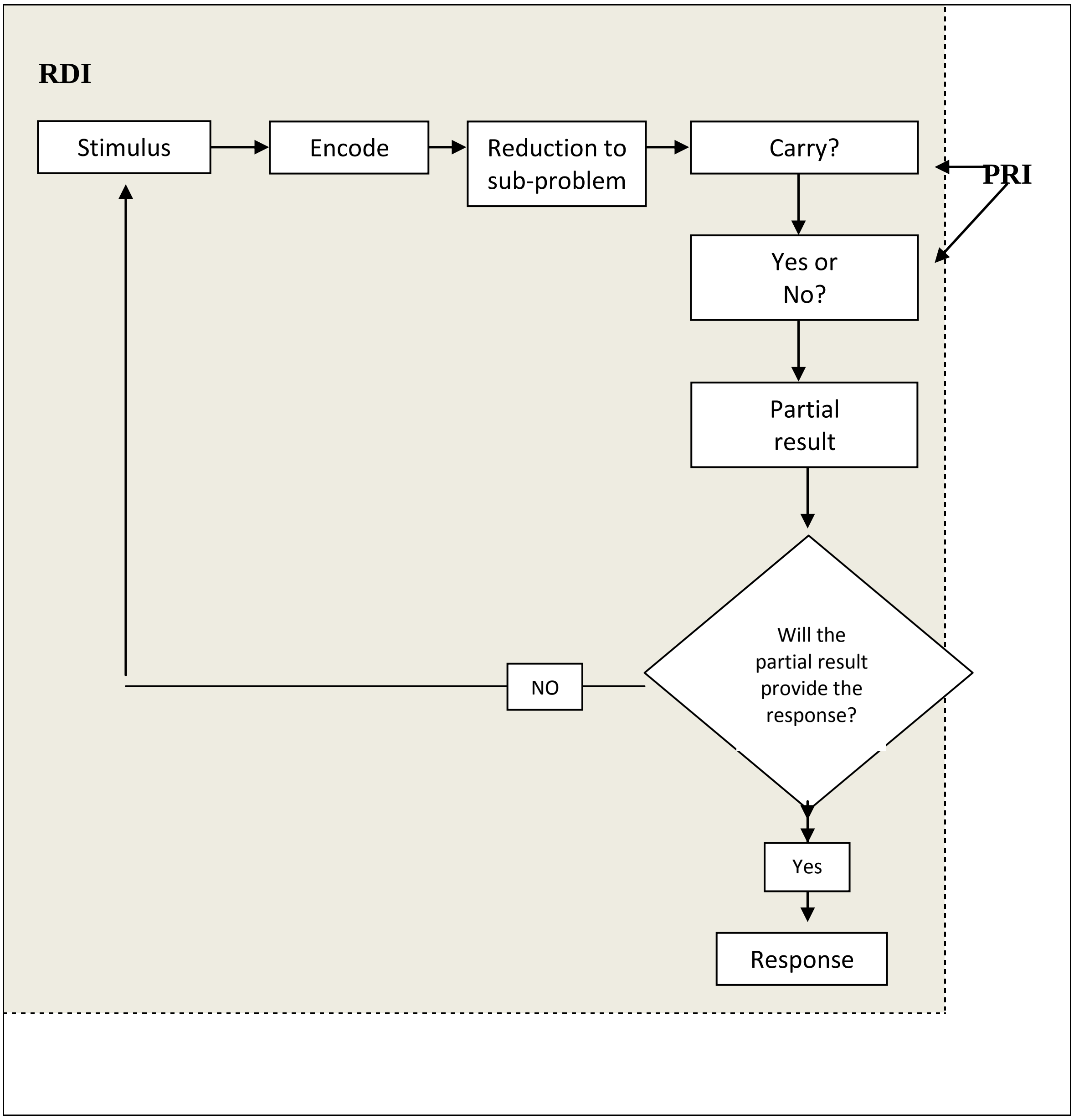


Figure 7.2. A Flow Diagram of the Division Process, Including the Inhibitory Mechanisms

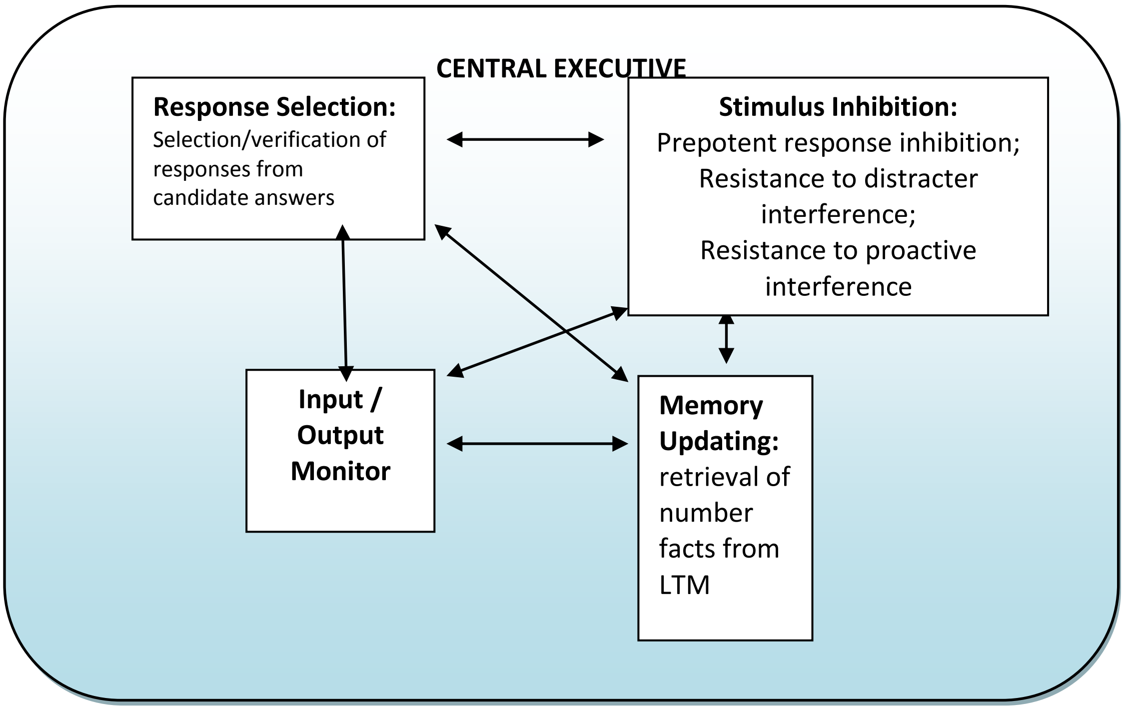
Friedman & Miyake (2004) describe PRI and RDI as a shared mechanism, Verbruggen (2005) suggested they formed a common inhibitory system and Brydges *et al* (2012) provides evidence for two separable sub-processors; all these are plausible notions. It is proposed, as a result of the finding from the present study that RDI and PRI form a two channel inhibitory system (see Figure 7.2). Note that RDI, as the proactive sub-component has been present throughout the calculation process throughout Experiment Two and during RDI load in Experiments Three and Four, hence its channel is coloured grey. PRI, as the reactive sub-component occupies the white area and therefore can enter the grey area whenever conflict is detected, consequently, the two channels are separated by a 'leak-prone' dotted line. In the diagram two arrows protrude from

PRI pointing to "Carry?" and "Yes or No?" The results of the present study suggest that these are the calculation procedures where conflict is most likely to manifest itself, in terms of the prepotent tendency not to carry (Fürst & Hitch, 2000; Imbo, Vandierendonck, & Vergauwe, 2007), or the possible zero/non-zero carry conflict evident from the results of Experiments Three and Four.

Two broad questions were asked at the beginning of this discussion: (1) Are prepotent response inhibition and resistance to distracter interference, as types of inhibition, utilised in the cognitive processing of complex division problems requiring and not requiring remainder-carrying procedures? (2) Are there any specific procedures, within the complex division process that benefit from the utilisation of these particular types of inhibition? To answer these questions, it is necessary to describe the division process in more detail starting with a two-carry problem, as if it were hidden amongst flanker digits. The stimulus represents the division problem, e.g., 1674 ÷ 3. The problem when inside flanker-digits triggers RDI, which persistently monitors for and prevents conflict. The problem is then encoded into 'sixteen-hundred-and- seventy-four divided by three.' The reduction represents (initially) 16 ÷ 3, this moves on to the "Carry?" stage where it is decided how many times 3 goes into 16 and whether there is any remainder to carry to the next digit. In this case, the intermediate result will be five which is encoded into 5 and the remainder is 1. Hence, '5 with 1 to carry' is stored and the stimulus is revisited. The stimulus can now be encoded into the second sub-problem, 17 ÷ 3. This is then stored as 5 with 2 to carry and the stimulus is revisited; the third sub-problem is 24 ÷ 3. The final part of the result is *exactly* 8, which after two approximate partial results may be questionable; hence PRI enters to prevent the possible interference of an approximate response.

In the case of a one carry problem, where the decision has to be made with regard to a carry operation, RDI itself intensifies to filter zero carry/ non-zero carry interference. PRI can remain inactivated if the intensity of RDI is sufficient to suppress this type of interference. It can also remain inactivated when problems demand no carries. In the case of no flanker digits being present, then no proactive conflict monitoring necessarily takes place.

It ought to be emphasised that the results of the present study provides no evidence that RDI is automatically employed by the cognitive processing system when solving division problems. What it does provide, however, is evidence that the employment of resistance to distracter interference, if it is used, is beneficial, in terms of accuracy. Prepotent response inhibition and resistance to distracter interference have been postulated as two out of three types of inhibitory processes that form a complete inhibitory mechanism (Friedman & Miyake, 2004). The complete mechanism can be considered akin to the stimulus inhibitor, a subcomponent of the central executive (Baddeley, 1996) which, in turn, form the attentional control component of working memory (Baddeley, 2000, 2003). As a result of the present research, a small amount of detail could be added to the latest four component (Baddeley, 2000) working memory model (see Figure 7.3).



Stimulus Inhibition
Resistance to Distracter interference: A proactive system for the prevention of interference, e.g., the filtration of zero/non-zero carry-values when the order is less predictable; maintenance of a particular goal such as a complex division problem to improve accuracy
Prepotent Response Inhibition: A reactive system to filter strong tendencies such as not to carry when carrying is required and to filter approximate responses after two such consecutive responses, when an exact response is required.
Resistance to Proactive Interference

Figure 7.3. A Diagram Displaying Enhancements to the Central Executive

Figure 7.3 provides a diagram displaying how the Central Executive of Baddeley's model of working memory may be enhanced by using derivations from the present study. Owing to the present study being focussed on mental arithmetic, examples of how each section work are arithmetically based. There has been controversy in the past over whether the central executive is nothing more than a homunculus or is really more of an executive committee (Baddeley, 1998, 2007; Parkin, 1998). It is proposed that from the results of the present study that one might take the notion of an executive committee a step further and refer also to subcommittees (only one subcommittee can be described in detail, here). The upper section of Figure 7.3 contains the central executive complete with the fractions proposed by Baddeley (1996). Inside the boxes are examples of arithmetic procedures that have been taken from previous literature (e.g., Deschuyteneer & Vandierendonck, 2005a, 2005b; De Rammelaere e*t al*, 2001; Deschuyteneer *et al*, 2006; Duverne *et al*, 2008; Fürst & Hitch, 2000; Imbo & Vandierendonck, 2007). Input monitoring does not seem to have any specific arithmetic rôles, as such (Deschuyteneer & Vandierendonck, 2005a, 2005b). The stimulus inhibition box in the upper diagram has been enlarged below and provides examples of rôles for RDI and PRI with regard to complex division, derived from the present research – more detail with regard to this is provided in the commentary to Figure7.2.

The arrows in the upper diagram denote the executive components working together. For example, if response selection is involved in selecting the appropriate response from several activated answers, then it may be that the inappropriate responses need to be inhibited (Deschuyteneer & Vandierendonck, 2005a, 2005b), hence the arrow between response selection and stimulus inhibition. The type of inhibition that would do this is not clear, but the evidence from the present study suggests that PRI and RDI is more likely to be involved in inhibiting inappropriate response-types (e.g., zero/ non-zero carries) rather than incorrect table-related responses. The type of inhibition that aids response selection (if it did need aiding) may speculatively be resistance to proactive interference.

The earlier reference to subcommittees is related to each of the subcomponents of the central executive: input monitoring, response selection, memory updating and stimulus inhibition. If these are four members of the executive committee, then, in the case of stimulus inhibition, PRI, RDI and resistance to proactive interference form an inhibitory sub-committee. Moreover, as has been proposed in the present study, two members of this subcommittee work particularly closely together.

Future Directions

It was stated earlier that some of the possible response-conflict may be owing to priming caused by previous responses to sub-problems, particularly in Experiments Three and Four. This type of priming may be more akin to proactive interference, i.e., interference of information that was relevant but has become irrelevant (Friedman & Miyake, 2004). Future research might carry out similar experiments to those in the present study but attempt to load the resistance to proactive interference system. One of the activities used by Friedman & Miyake (2004) was the Brown-Peterson Task (Kane & Engle, 2000) where participants were instructed to learn word lists and recall those from the same category. If this were used as a concurrent task within the context of mental arithmetic, it may not be effective in creating an appropriate load. One method might be to display the response to the previous problem simultaneously with the present problem. This might cause proactive interference.

Another point of interest in the present study was the extraction of the carrying and the short division procedures from the complex division process. As far as is known, at the time of writing, this had not been done before and provided insights, for example, into the rôles of PRI and RDI in resolving zero/non-zero response conflict; this is one example where such phenomena would not have been discovered without extracting procedures. Besides the two procedures extracted in Experiments Three and Four there is the planning procedure as to the methodology to be used to solve the problem, the *decision making procedure* as to whether or not to carry, and the *subtraction procedure* in order to decide on the value of the carry, where it is needed. It has been seen from the results of Experiment Four that there was a mild speed accuracy trade-off under RDI manipulation but only when two carries were required. This did not fully explain the speed-accuracy trade-offs evident in Experiment Two. The latter two procedures mentioned might be easily extracted using similar methods to those employed in Experiments Three and Four. This might provide more insight into the rôles of prepotent response inhibition and particularly resistance to distracter interference in carrying out these individual procedures. No doubt, this list of procedures is not exhaustive and, if ways could be found of extracting these for further study, this would provide more opportunity for future research.

Experiment Three provided the opportunity to briefly examine types of responses. There was a significant effect of PRI and RDI in terms of latencies when complete problems required two carries and the sub-problems were *divisible*. From the results of this experiment, it was proposed that this was owing to two consecutive non-divisible sub-problems followed by one divisible sub-problem and therefore PRI was activated to resolve the resulting non-divisible/divisible conflict. Future research might examine this phenomenon by creating further sets of division problems that demand 4, 5 or 6 digit responses, using the number of digits as an independent variable and examining the results under different carry conditions. One might then ascertain whether the same phenomenon occurs after more consecutive approximate responses, whether this would happen in the middle or at the beginning of the complete problem, as well as under different types of inhibition, hence one might discover whether two consecutive response-types is enough to create a prepotent tendency. Related to this but suggesting opposing results, there is evidence that approximate responses use *less* working memory resources than exact responses within the context of simple addition, however, rounding down takes more working memory resources than rounding up (Kalaman & LeFevre, 2007). Furthermore it has been proposed that approximation may depend on executive resources, for example a problem such as 32 + 39 may be approximated to 30 + 40 = 70 and then the exact response selected from 69, 71 or 72 (Logie, *et al*, 1994); more specifically, the selection of the correct response may be a rôle for response selection (cf. Deschuyteneer & Vandierendonck, 2005a, 2005b). In the present study, because *integer* values were sought from participants, all responses for non-divisible sub-problems involved rounding down, and were within the context of e.g., 'the number of times 6 divides into 19' rather than approximating 32 + 39 and probably calls upon different working memory resources. From examining the secondary task used by Kalaman & LeFevre (2007), it is likely that suppression was aimed at the visuo-spatial sketchpad (they asked participants to remember a string of four letters whilst responding to arithmetic tasks). There seems to be a lack of literature aimed at studying executive abilities with regard to arithmetic approximation and different types of approximation. The suggested future research may help to clarify this very indirect mismatch of response-type behaviour.

The present study has provided a method of separating the prepotent response inhibition system from Baddeley's fractionated model of working memory (1996). This method of inducing a particular response situation and then stopping it but leaving the temptation to continue the response situation might be extended, as a type of methodology, in the future for other behavioural experiments. The Eriksen Flanker Task (Eriksen & Eriksen, 1974) had already undergone extensions into numerical work, mainly digit recognition. For the present study, it was a case of extending it considerably further and utilising the top-heavy fraction format of the division problems. There is probably room for further extension of this task for future research.

There are, no doubt, many more procedures involved within the complex division process that may be extractable; the short-division and carrying procedures are just two of them. The above suggestions might help to discover which components and subcomponents might be involved in the many procedures comprising the complete division process. Research into working memory has made great strides since the original three-component proposal of working memory was postulated (Baddeley & Hitch, 1974). These strides have included the fractionation of the central executive (Baddeley, 1996) and further separation of the fraction, in relation to the present study, the most relevant being the separation of stimulus inhibition into PRI, RDI and RPI. The present research has led to the proposal that PRI and RDI form a two-channel common inhibitory system. What has not been answered is the question with regard to the function of resistance to proactive interference and how it might relate to other executive components and subcomponents.

Looking to the future, one might speculate that, if a more direct matching can be suggested between executive functions or abilities and arithmetic procedures then both children and adults who display arithmetic difficulties might eventually be able to be aided by, what might be termed, ‘executive function improvement programmes,’ which may, in turn, help to alleviate at least some arithmetic difficulties. This might be deemed controversial, particularly if there is not complete agreement regarding a causal link between executive control and weak arithmetic skills. What ought to be less controversial is that unless further direct evidence can be unearthed that executive control is actually employed by the cognitive processing system within the context of mental arithmetic, it will remain controversial as to whether any executive function improvement programmes may be of any benefit. The present study has suggested that prepotent response inhibition is involved in a small way and resistance to distracter interference, if it is employed, is beneficial in terms of accuracy. This suggested ‘partnership’ between cognitive psychology and remedial mathematical education may seem a ‘tall order’ but, unlike the case of reading difficulties, many people are not ashamed to admit to having arithmetic difficulties of one type or another, it is just accepted whether it ought to be or not.

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Appendix I Experimental Problems (Experiments 1 and 2)

Group 1				Group 2 (PRI)				Group 3			
	No Carry	**Single Carry**	Two Carries		No Carry	**Single Carry**	Two Carries		No Carry	**Single Carry**	Two Carries
1.	1264 2	**1452 2**	1536 2	***1.***	1648 2	**1674 2**	1358 2	***1.***	1848 2	**1524 2**	1732 2
2.	1596 3	**1242 3**	1725 3	***2.***	1839 3	**1581 3**	1344 3	***2.***	1563 3	**1875 3**	1422 3
3.	1688 4	**1292 4**	1348 4	***3.***	1284 4	**1656 4**	1456 4	***3.***	1608 4	**1480 4**	1336 4
4.	1555 5	**1575 5**	1765 5	***4.***	1055 5	**1095 5**	1675 5	***4.***	1550 5	**1650 5**	1865 5
5.	1866 6	**1284 6**	1344 6	***5.***	1266 6	**1926 6**	1458 6	***5.***	1206 6	**1326 6**	1464 6
6.	1477 7	**1491 7**	1624 7	***6.***	1470 7	**1617 7**	1673 7	***6.***	1407 7	**1757 7**	1764 7
7.	1688 8	**1696 8**	1856 8	***7.***	1600 8	**1768 8**	1784 8	***7.***	1680 8	**1848 8**	1936 8
8.	1899 9	**1989 9**	1917 9	***8.***	1890 9	**1989 9**	1926 9	***8.***	1800 9	**1980 9**	1908 9

9.	1428 2	**1656** **4**	1452 6	***9.***	1806 6	**1346** **2**	1174 2	***9.***	1824 2	**1326** **6**	1536 6
10.	1293 3	**1570** **5**	1568 7	***10.***	1470 7	**1719** **3**	1944 3	***10.***	1539 3	**1757** **7**	1701 7
11.	1248 4	**1872** **6**	1784 8	***11.***	1608 8	**1848** **8**	1744 4	***11.***	1505 5	**1696** **8**	1944 8
12.	1055 5	**1484** **7**	1998 9	***12.***	1809 9	**1989** **9**	1725 5	***12.***	1684 4	**1818** **9**	1926 9

Group 2 (RDI)			
	No Carry	**Single Carry**	Two Carries
1.	7771648777 7772777	**8881674888** **8882888**	4441358444 4442444
2.	4441839444 4443444	**4441581444** **4443444**	7771344777 7773777
3.	3331284333 3334333	**2221656222** **2224222**	7771456777 7774777
4.	7771055777 7775777	**7771095777** **7775777**	8881675888 8885888
5.	7771266777 7776777	**7771926777** **7776777**	7771458777 7776777

6.	3331470333 3337333	**8881617888 8887888**	4441673444 4447444
7.	2221600222 2228222	**4441768444 4448444**	2221784222 2228222
8.	7771890777 7779777	**7771989777 7779777**	7771926777 7779777
9.	2221806222 2226222	**7771346777 7772777**	3331174333 3332333
10.	3331470333 3337333	**4441719444 4443444**	7771944777 7773777
11.	2221608222 2228222	**7771848777 7778777**	8881744888 8884888
12.	2221809222 2229222	**7771989777 7779777**	8881725888 8885888

Appendix II Random Order of Division Problems

Group 1 (Simultaneous) (Experiment 1 only)	Group 2 (Inhibition – Exp. 1 / RDI Exp. 2)	Group 3 (Control)
G1/1a) 1264 2	G2/3c) 1456 4	G3/3a 1608 4
G1/5a) 1866 6	G2/5c) 1458 6	G3/1c 1732 2
G1/6b) 1491 7	G2/1b) 1674 2	G3/5b 1326 6
G1/4c) 1765 5	G2/8c) 1926 9	G3/8b 1980 9
G1/7a) 1688 8	G2/5b) 1926 6	G3/10b 1757 7
G1/3b) 1292 4	G2/ 2c) 1344 3	G3/1a 1848 2
G1/8b) 1989 9	G2/8a) 1890 9	G3/7c 1936 8
G1/6c) 1628 7	G2/7b) 1768 8	G3/4a 1550 5
G1/4a) 1555 5	G2/2a) 1839 3	G3/11b 1696 8
G1/6a) 1477 7	G2/2b) 1581 3	G3/3b 1480 4
G1/1c) 1536 2	G2/7c) 1784 8	G3/9c 1536 6
G1/4b) 1575 5	G2/3a) 1284 4	G3/6a 1407 7
G1/3a) 1688 4	G2/4b) 1095 5	G3/8c 1908 9
G1/ 7c) 1856 8	G2/1c) 1358 2	G3/9a 1824 2
G1/2b) 1242 3	G2/6a) 1477 7	G3/2b 1875 3

G1/2a) 1596 3	G2/4a) 1055 5	G3/10c 1701 7
G1/7b) 1696 8	G2/6c) 1673 7	G3/1b 1524 2
G1/8a) 1899 9	G2/8b) 1989 9	G3/10a 1539 3
G1/2c) 1725 3	G2/3b) 1656 4	G3/12b 1818 9
G1/5b) 1284 6	G2/7a) 1688 8	G3/6c 1764 7
G1/8c) 1917 9	G2/4c) 1675 5	G3/3c 1336 4
G1/1b) 1452 2	G2/6b) 1617 7	G3/11a 1505 5
G1/5c) 1344 6	G2/5a) 1266 6	G3/7b 1848 8
G1/3c) 1348 4	G2/1a) 1648 2	G3/5a 1206 6
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G1/10a) 1293 3	G2/10a) 1470 7	G3/11c 1944 8
G1/9b) 1656 4	G2/9b) 1346 2	G3/2c 1422 3
G1/10c) 1568 7	G2/10c) 1944 3	G3/6b 1757 7
G1/11b) 1872 6	G2/11b) 1848 8	G3/9b 1326 6
G1/9a) 1428 2	G2/9a) 1806 6	G3/4c 1865 5
G1/9c) 1452 6	G2/9c) 1174 2	G3/8a 1800 9
G1/11c) 1784 8	G2/11a) 1608 8	G3/12c 1926 9
G1/11a) 1248	G2/11c) 1744 4	G3/7a 1680

4 G1/12a) 1055 5 G1/12b) 1484 7 G1/12c) 1998 9 G1/10b) 1570 5	G2/12a) 1809 9 G2/10b) 1719 3 G2/12b) 1989 9 G2/12c) 1725 5	8 G3/5c 1464 6 G3/12a 1684 4 G3/2a 1563 3 G3/4b 1650 5

Appendix III Experimental Problems (Experiments 3 and 4)

Group 1				Group 2 (PRI)				Group 3			
	No Carry	**Single Carry**	Two Carries		No Carry	**Single Carry**	Two Carries		No Carry	**Single Carry**	Two Carries
1.	1264 2 12 6 4 2	**1452 2 14 5 12 2**	1536 2 15 13 16 2	***1.***	1648 2 16 4 8 2	**1674 2 16 7 14 2**	1358 2 13 15 18 2	***1.***	1848 2 18 4 8 2	**1524 2 15 12 4 2**	1732 2 17 13 12 2
2.	1596 3 15 9 6 3	**1242 3 12 4 12 3**	1725 3 17 22 15 3	***2.***	1839 3 18 3 9 3	**1581 3 15 8 21 3**	1344 3 13 14 24 3	***2.***	1563 3 15 6 3 3	**1875 3 18 7 15 3**	1422 3 14 22 12 3
3.	1688 4 16 8 8 4	**1292 4 12 9 12 4**	1348 4 13 14 28 4	***3.***	1284 4 12 8 4 4	**1656 4 16 5 16 4**	1456 4 14 25 16 4	***3.***	1608 4 16 0 8 4	**1480 4 14 28 0 4**	1336 4 13 13 16 4
4.	1555 5 15 5 5 5	**1575 5 15 7 25 5**	1765 5 17 26 15 5	***4.***	1055 5 10 5 5 5	**1095 5 10 9 45 5**	1675 5 16 17 25 5	***4.***	1550 5 15 5 0 5	**1650 5 16 15 0 5**	1865 5 18 36 15 5
5.	1866 6 18 6 6	**1284 6 12 8 24**	1344 6 13 14 24	***5.***	1266 6 12 6 6	**1926 6 19 12 6**	1458 6 14 25 18	***5.***	1206 6 12 0 6	**1326 6 13 12 6**	1464 6 14 26 24

	6	**6**	6		6	**6**	6		6	**6**	6
6.	$\frac{1477}{7}$ $\frac{14\ 7\ 7}{7}$	$\mathbf{\frac{1491}{7}}$ $\mathbf{\frac{14\ 9\ 21}{7}}$	$\frac{1624}{7}$ $\frac{16\ 22\ 14}{7}$	***6.***	$\frac{1470}{7}$ $\frac{14\ 7\ 0}{7}$	$\mathbf{\frac{1617}{7}}$ $\mathbf{\frac{16\ 21\ 7}{7}}$	$\frac{1673}{7}$ $\frac{16\ 27\ 63}{7}$	***6.***	$\frac{1407}{7}$ $\frac{14\ 0\ 7}{7}$	$\mathbf{\frac{1757}{7}}$ $\mathbf{\frac{17\ 35\ 7}{7}}$	$\frac{1764}{7}$ $\frac{17\ 36\ 14}{7}$
7.	$\frac{1688}{8}$ $\frac{16\ 8\ 8}{8}$	$\mathbf{\frac{1696}{8}}$ $\mathbf{\frac{16\ 9\ 16}{8}}$	$\frac{1856}{8}$ $\frac{18\ 25\ 16}{8}$	***7.***	$\frac{1600}{8}$ $\frac{16\ 0\ 0}{8}$	$\mathbf{\frac{1768}{8}}$ $\mathbf{\frac{17\ 16\ 8}{8}}$	$\frac{1784}{8}$ $\frac{17\ 18\ 24}{8}$	***7.***	$\frac{1680}{8}$ $\frac{16\ 8\ 0}{8}$	$\mathbf{\frac{1848}{8}}$ $\mathbf{\frac{18\ 24\ 8}{8}}$	$\frac{1936}{8}$ $\frac{19\ 13\ 56}{8}$
8.	$\frac{1899}{9}$ $\frac{18\ 9\ 9}{9}$	$\mathbf{\frac{1989}{9}}$ $\mathbf{\frac{19\ 18\ 9}{9}}$	$\frac{1917}{9}$ $\frac{19\ 11\ 27}{9}$	***8.***	$\frac{1890}{9}$ $\frac{18\ 9\ 0}{9}$	$\mathbf{\frac{1989}{9}}$ $\mathbf{\frac{18\ 18\ 9}{9}}$	$\frac{1926}{9}$ $\frac{19\ 12\ 36}{9}$	***8.***	$\frac{1800}{9}$ $\frac{18\ 0\ 0}{9}$	$\mathbf{\frac{1980}{9}}$ $\mathbf{\frac{19\ 18\ 0}{9}}$	$\frac{1908}{9}$ $\frac{19\ 10\ 18}{9}$

Group 2 (RDI)			
	No Carry	**Single Carry**	Two Carries
1.	$777\underline{1648}777$ $777\underline{2}777$ $\frac{16\ 4\ 8}{2}$	$\mathbf{888\underline{1674}888}$ $\mathbf{888\underline{2}888}$ $\mathbf{\frac{16\ 7\ 14}{2}}$	$444\underline{1358}444$ $444\underline{2}444$ $\frac{13\ 15\ 18}{2}$
2.	$444\underline{1839}444$ $444\underline{3}444$	$\mathbf{444\underline{1581}444}$ $\mathbf{444\underline{3}444}$	$777\underline{1344}777$ $777\underline{3}777$

	$\frac{18\;\;3\;\;9}{3}$	$\boldsymbol{\frac{15\;\;8\;\;21}{3}}$	$\frac{13\;\;14\;\;24}{3}$
3.	$333\underline{1284}333$ $333\underline{4}333$ $\frac{12\;\;8\;\;4}{4}$	$\mathbf{222\underline{1656}222}$ $\mathbf{222\underline{4}222}$ $\boldsymbol{\frac{16\;\;5\;\;16}{4}}$	$777\underline{1456}777$ $777\underline{4}777$ $\frac{14\;\;25\;\;16}{4}$
4.	$777\underline{1055}777$ $777\underline{5}777$ $\frac{10\;\;5\;\;5}{5}$	$\mathbf{777\underline{1095}777}$ $\mathbf{777\underline{5}777}$ $\boldsymbol{\frac{10\;\;9\;\;45}{5}}$	$888\underline{1675}888$ $888\underline{5}888$ $\frac{16\;\;17\;\;25}{5}$
5.	$777\underline{1266}777$ $777\underline{6}777$ $\frac{12\;\;6\;\;6}{6}$	$\mathbf{777\underline{1926}777}$ $\mathbf{777\underline{6}777}$ $\boldsymbol{\frac{19\;\;12\;\;6}{6}}$	$777\underline{1458}777$ $777\underline{6}777$ $\frac{14\;\;25\;\;18}{6}$
6.	$333\underline{1470}333$ $333\underline{7}333$ $\frac{14\;\;7\;\;0}{7}$	$\mathbf{888\underline{1617}888}$ $\mathbf{888\underline{7}888}$ $\boldsymbol{\frac{16\;\;21\;\;7}{7}}$	$444\underline{1673}444$ $444\underline{7}444$ $\frac{16\;\;27\;\;63}{7}$
7.	$222\underline{1600}222$ $222\underline{8}222$ $\frac{16\;\;0\;\;0}{8}$	$\mathbf{444\underline{1768}444}$ $\mathbf{444\underline{8}444}$ $\boldsymbol{\frac{17\;\;16\;\;8}{8}}$	$222\underline{1784}222$ $222\underline{8}222$ $\frac{17\;\;18\;\;24}{8}$

8.	777<u>1890</u>777 777<u>9</u>777 <u>18 9 0</u> 9	**777<u>1989</u>777** **777<u>9</u>777** **<u>18 18 9</u>** **9**	777<u>1926</u>777 777<u>9</u>777 <u>19 12 36</u> 9

Note: Upper problems are complete problems; lower problems are sub-problems, e.g., for the PRI condition, 1648/2 was presented as

<u>16</u> ***<u>4</u>*** ***<u>8</u>***
<u>2</u> followed by <u>2</u> followed by <u>2</u>, whereas

for the RDI condition, 777<u>1648</u>777 was presented as
777<u>2</u>777

777<u>16</u>777 followed by 777<u>4</u>777 followed by 777<u>8</u>777
777<u>2</u>777 777<u>2</u>777 777<u>2</u>777.

Complete problems were not displayed for Experiments 3 and 4.

Appendix IV

Experiment Three

A Check on the Manipulative Effects of the Simultaneous Activity: RTs

Table 1.

Control vs. Simultaneous RTs

	Mean	S D	N
Mean: Control (0 carries)	1182.64	272.524	36
Mean: Control (1 carry)	1804.03	476.543	36
Mean: Control (2 carries)	2488.12	734.406	36
Mean: Simultaneous (0 carries)	1903.57	497.378	36
Mean: Simultaneous (1 carry)	2232.10	626.329	36
Mean: Simultaneous (2 carries)	2813.76	986.926	36

As a manipulation check, a 2(control [problems] vs. simultaneous) x 3 (0 vs. 1vs. 2 carries) repeated-measures ANOVA on the control vs. simultaneous conditions (RTs) disclosed a significant main effect of the simultaneous activity, $F(1, 35) = 46.89$, $p < 0.001$, $\eta^2_p = 0.57$ and of the number of carries, $F(2, 70) = 131.83$, $p < 0.001$, $\eta^2_p = 0.79$, reflecting the overall increase in latencies as en effect of the simultaneous load. There was also a significant cognitive factor x arithmetic factor interaction, $F(2, 70) = 12.13$, $p < 0.001$, $\eta^2_p = 0.26$, reflecting the steeper increase under no carries.

A Check on the Manipulative Effects of the Simultaneous Activity: Error Rates

A 2 (control vs. simultaneous) x 3 (0 vs. 1 vs. 2 carries) repeated-measures ANOVA disclosed only one significant main effect: of arithmetic condition, $F(2, 70) = 18.52$, $p < 0.001$, $\eta^2_p = 0.35$. The effect of the cognitive condition did not reach significance, $F = 0.002$, $p > 0.05$, neither did the cognitive-condition x arithmetic condition interaction, F = 1.17, p > 0.05. The error rate was low.

The Effects of Arithmetic Conditions on Direction Stating (RTs)

A single factor (control vs. 0 carries cs. 1 carry vs. 2 carries) ANOVA revealed a significant effect of arithmetic factor $F(3, 105) = 4.21$, p =0.007, $\eta^2_p = 0.11$. A series of one-tailed t-tests revealed three significant comparisons: control (direction) vs. no-carries (direction), $t(35) = -10.62$, $p < 0.001$, $\alpha = 0.0167$; control (direction) vs. one-carry (direction), $t(35) = -2.43$, $p = 0.001$, $\alpha = 0.05$; control (direction) vs. two-carries (direction), $t(35) = -9.77$, $p < 0.001$, $\alpha = 0.033$, reflecting significant slowing of direction stating latencies when having to solve problems and say, “left / right,” simultaneously.

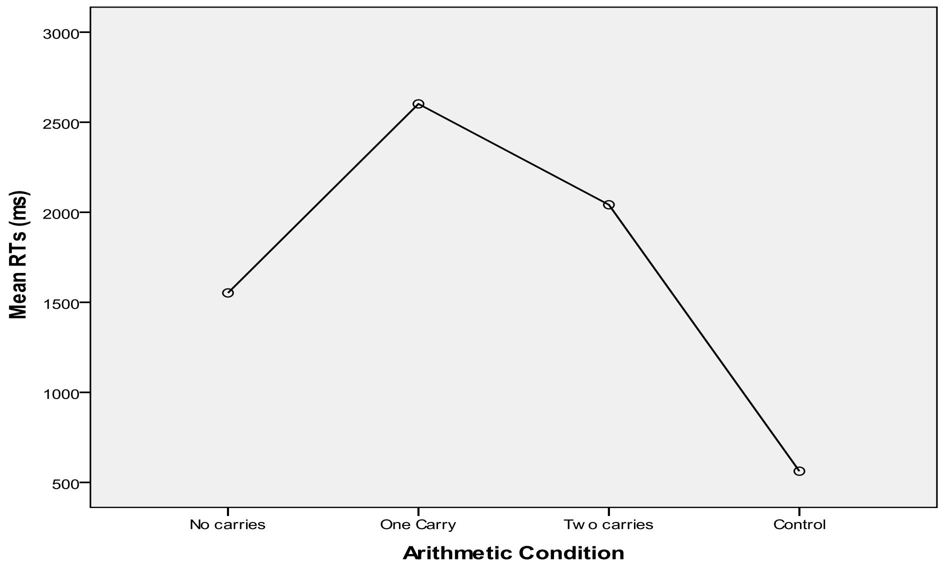

Figure 1. *Mean RTs: Direction Stating*

Errors in Direction Stating: Simultaneous and Prepotent Inhibition Conditions Compared

The errors in direction stating were few and spread amongst a small number of participants. As these were most unlikely to follow even an approximate parametric pattern, a series of Related Samples Wilcoxon Signed Ranks Tests were carried out and revealed one significant difference.

The Direction Errors (Simultaneous – two-carries) versus the Direction Errors (Prepotent Response Inhibition – two-carries) comparison generated a significant difference, $p = 0.007$ suggesting that the simultaneous condition disturbed direction-stating more than the PRI condition did, most likely because of its heavier load on the phonological system. None of the other comparisons reached significance, $p > 0.05$.

Appendix V

Experiment Four

A Check on the Manipulation Activity (Simultaneous Condition) - RTs

The simultaneous condition, being a manipulative condition, was analysed separately. These were of less interest in this experiment owing to the multiple elements of working memory it was thought to load. However, to have the desired effect of inducing a prepotent response, it probably ought to be demonstrated that the simultaneous condition had at least some effect. A 2 (control vs. simultaneous) x 2 (1 carry vs. 2 carries) ANOVA revealed a significant main effect of the simultaneous activity, $F(1, 35) = 27.03$, $p < 0.001$, $\eta^2_p = 0.44$ and of the number of carries, $F(1, 35) = 14.86$, $p < 0.001$, $\eta^2_p = 0.30$. There was no significant cognitive-factor x arithmetic-factor interaction, $F(1 ,35) = 0.73$, $p = 0.40$. Two one-tailed *post-hoc* tests confirmed that the main effect of the simultaneous condition was to cause a significant increase in latencies: one carry, $t(35) = -4.91$, $p < 0.001$; and two carries, $t(35) = -4.41$, $p < 0.001$.

The Effect of the Manipulative Activity on Direction-Stating RTs

Table 1.

Mean Latencies (Control and Simultaneous Conditions)

Condition		Mean RTs (ms)	SD
Control		579	132
Simultaneous	One Carry	1520	568
	Two Carries	1580	723

An AVOVA was carried out on the RTs of the direction stating (left / right) and indicated a significant main effect of arithmetic condition, $F(2, 70) = 63.95$, $p < 0.001$, $\eta^2_p = 0.65$. Three one tailed t-tests suggested a significant increase in latencies between control and one carry, $t(35) = -10.11$, $p < 0.001$, and between control and two carries, $t(35) = -8.39$, $p < 0.001$, indicating that solving the sub-problems did have a slowing effect on saying, “left / right.”

The Effect of the Manipulative Activity on Errors

The Error rate regarding the division problems was subjected to an ANOVA but revealed no significant main effects or interactions. A pair of *post-hoc* tests revealed no significant increase in error rates from the control to the simultaneous conditions, $p > 0.005$.

The ‘direction-stating’ errors under the simultaneous condition were few and far between and the distribution pattern was assumed to be non-parametric, hence a series of Related Samples Wilcoxon Signed Rank Tests were carried out and generated the results in Table 2. These indicated that the simultaneous activity was far more taxing than the inhibition activity.

Table 2.

Results of Wilcoxon Signed Rank Tests (One-Sided) N= 36

Conditions Compared	*p*-value	Significant?
Control (Direction) vs. Prepotent Response Inhibition (1 carry)	0.090	No
Control (Direction) vs. Prepotent Response Inhibition (2 carries)	0.785	No
Control (Direction) vs. Simultaneous (1 carry)	0.004	Yes
Control (Direction) vs. Simultaneous (2 carries)	0.021	Yes
Prepotent Resp. Inhibition (1 carry) vs. Simultaneous (1 carry)	0.004	Yes
Prepotent Resp. Inhibition (2 carries) vs. Simultaneous (2 carries)	0.17	No